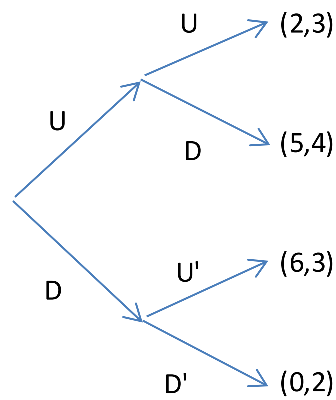
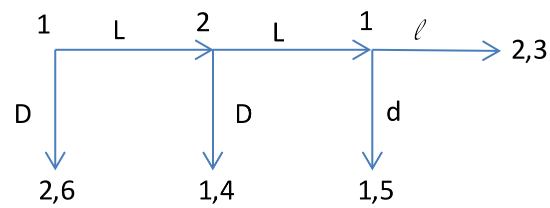
Name:_____________ KEY _________

1. (5 pts.) Prepackaged Game 1



Solve the above game, write the subgame perfect equilibrium, and the equilibrium payoffs.

2. (5 pts.) Prepackaged Game 2



Solve the above game, write the subgame perfect equilibrium, and the equilibrium payoffs.

3. Strategic Voting

Consider three board members of a company, Alice, Bob, and Ted, who are voting on whether to give themselves a pay raise. The raise is worth *b*, but each board member who votes for the raise incurs a cost, *c*, of union resentment and political backlash. Also, $c < b$. The outcome is decided by majority rule. If the vote fails, then everyone gets zero. Alice votes first, then Bob sees Alice's choice and votes, and finally Ted sees both Alice's and Bob's choice and votes.

a) (5 pts.) Draw and clearly label the game.
b) (5 pts.) Solve for the Subgame Perfect Equilibrium.
c) (5 pts.) Was there any advantage to moving first or last in this game? Discuss and explain your answer.

4. Limit Pricing

A firm, Player 1, is considering entering, E, a market currently dominated by a monopolist, Player 2. If P1 enters, P2 can either cooperate, C, with P1 or fight, F. If P1 doesn't enter, N, then it earns zero and the monopolist earns monopoly profits. If P1 does enter, E, it must pay an entry cost, C = 10, and faces two different possible scenarios:

Scenario 1: If the two firms cooperate, C, they both charge the same price and split the market evenly. In particular they produce the same total market quantity as the monopolist did, Q^M, but each produces half, $Q^M = q_1^D + q_2^D$. This means that that consumers pay the same under the monopolist and under the duopoly when they cooperate, $P^M = P_1^D = P_2^D$. P2 will earn half its monopoly profits. P1 will earn the same minus the entry cost, C.

Scenario 2: The incumbent, P2, fights by pricing at marginal cost. Assume both firms have constant but different same marginal costs where the entrant has slightly lower MC than the monopolist.

The following equations describe this situation:

No-Entry & Monopoly profits:
$$\pi_1 = 0$$
$$\pi_2^M = \left(P^M - mc^M\right)Q^M$$

Cooperative profits:
$$\pi_1^D = \left(P^D - mc_1\right)q_1^D - C$$
$$\pi_2^D = \left(P^D - mc_2\right)q_2^D$$

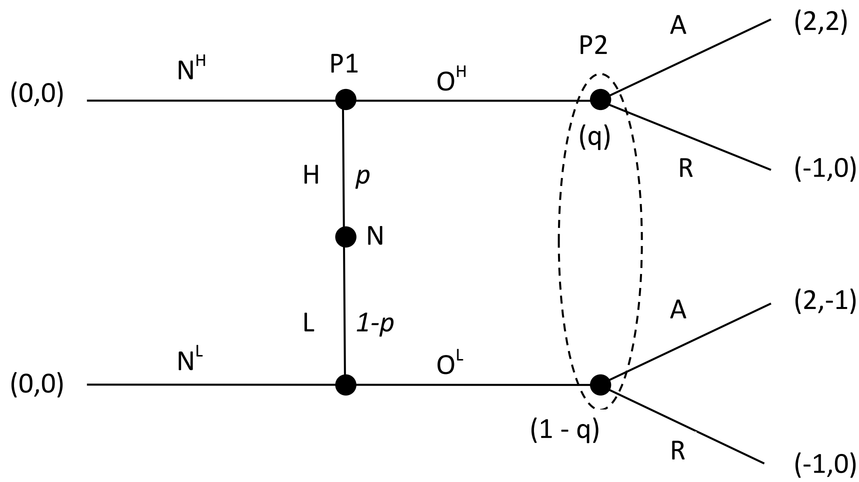
Fight profits:
$$\pi_1^F = \left(P_1^F - mc_1\right)q_1^F - C$$
$$\pi_2^F = \left(P_2^F - mc_2\right)q_2^F$$

Market Demand curve: $P = 300 - 2Q$

Constant (but different) Marginal costs:
$$mc_1 = 80$$
$$mc_2 = \$100$$

a) (10 pts.) Determine Q, P, and profits for each player in each situation. Be sure to show your work and how your determined these values.
b) (5 pts.) Draw and clearly label the game and solve for the Subgame Perfect Equilibrium.
c) (10 pts.) Was the monopolist's threat to "fight" credible? Based on our discussion from class, what additional strategies might allow the incumbent to make a credible threat? Discuss and explain clearly.

5. Firm Signaling Game

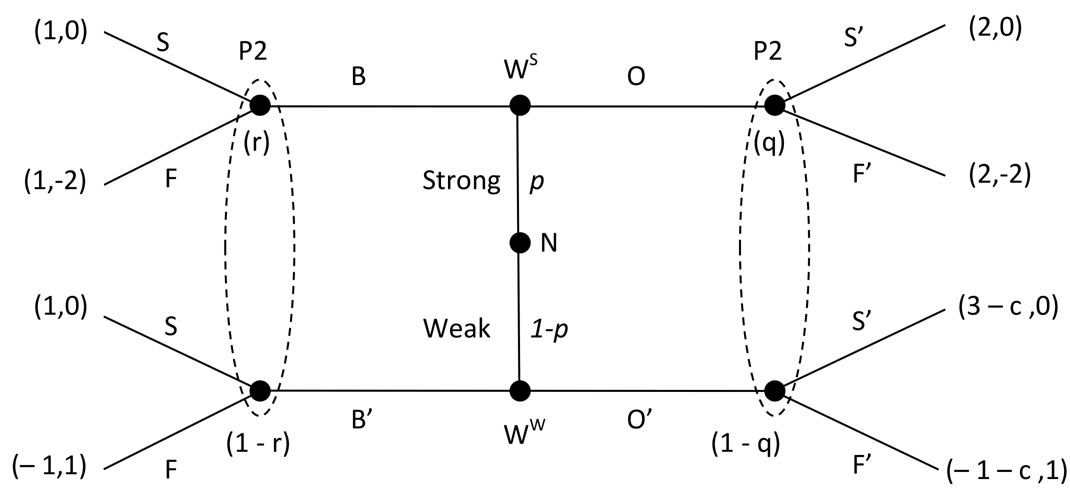


Consider the extensive-form game of incomplete information depicted above. There is a firm and a worker. In this game, nature first chooses the “type” of the firm (player 1). With probability p, the firm is of high quality (H) and, with probability 1 – p, the firm is of low quality (L). The firm chooses either to offer a job to the worker (O) or not to offer a job (N). If no job is offered, the game ends and both parties receive 0. If the firm offers a job, then the worker either accepts (A) or rejects (R) the offer. The worker’s effort on the job brings the firm a profit of 2. If the worker rejects an offer of employment, then the firm gets a payoff of -1 (associated with being “jilted”). Rejecting an offer yields a payoff of 0 to the worker. Accepting an offer yields the worker a payoff of 2 if the firm is of high quality and -1 if the firm is of low quality. The worker does not observe the quality of the firm directly.

a. (5 pts.) Check all *separating* equilibria. Does this game have a *separating* perfect Bayesian equilibrium? If so, fully describe it.
b. (5 pts.) Check all *pooling* equilibria. Does this game have a *pooling* perfect Bayesian equilibrium? If so, fully describe it.

6. The Princess Bride Game

In the classic Rob Reiner movie The Princess Bride, there is a scene at the end where Wesley (the protagonist) confronts the evil prince Humperdinck. The interaction can be modeled as the following game. Wesley is one of two types: weak or strong. Wesley knows whether he is weak or strong, but the prince only knows that he is weak with probability *p* and strong with probability *1-p*. Wesley is lying in bed in the prince's castle when the prince enters the room. Wesley decides whether to get out of bed (O) or stay in bed (B). The prince observes Wesley's action but does not observe Wesley's type. The prince then decides whether to fight (F) or surrender (S) to Wesley. The payoffs are such that the prince prefers to fight only with the weak Wesley, because otherwise the prince is an inferior swordsman. Also, the weak Wesley must pay a cost c to get out of bed. The extensive-form representation of the game is:



a. (10 pts.) Check all *separating* equilibria. What conditions on *c* and *p* guarantee the existence of a *separating* equilibrium? Fully describe such an equilibrium.
b. (10 pts.) Check all *pooling* equilibria. For what values of *c* and *p* is there *a pooling equilibrium* in which both strong and weak Wesley get out of bed? Fully describe such an equilibrium.

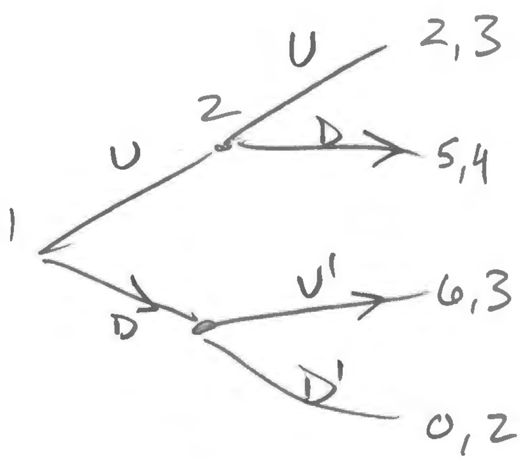
7. Consider the Spence Education Game from class.

(Discuss and explain your answers clearly and completely)

a. (10 pts.) What is the fundamental problem the employer faces in this game? How does the employer's problem change with the proportion of types in the general population? How high would the proportion of high types have to get to change the game's equilibrium? Does that make sense? Explain.
b. (10 pts.) What makes the signal work in this game? How high does the educational cost have to rise to change the game's equilibrium? Does that make sense? Explain.

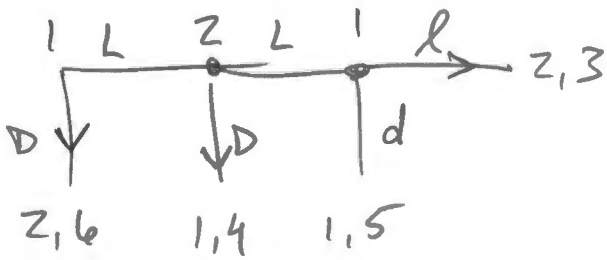
Final Key 1

#1



SPE:
P1 chooses D
P2 chooses DU'
payoffs (6,3)

#2

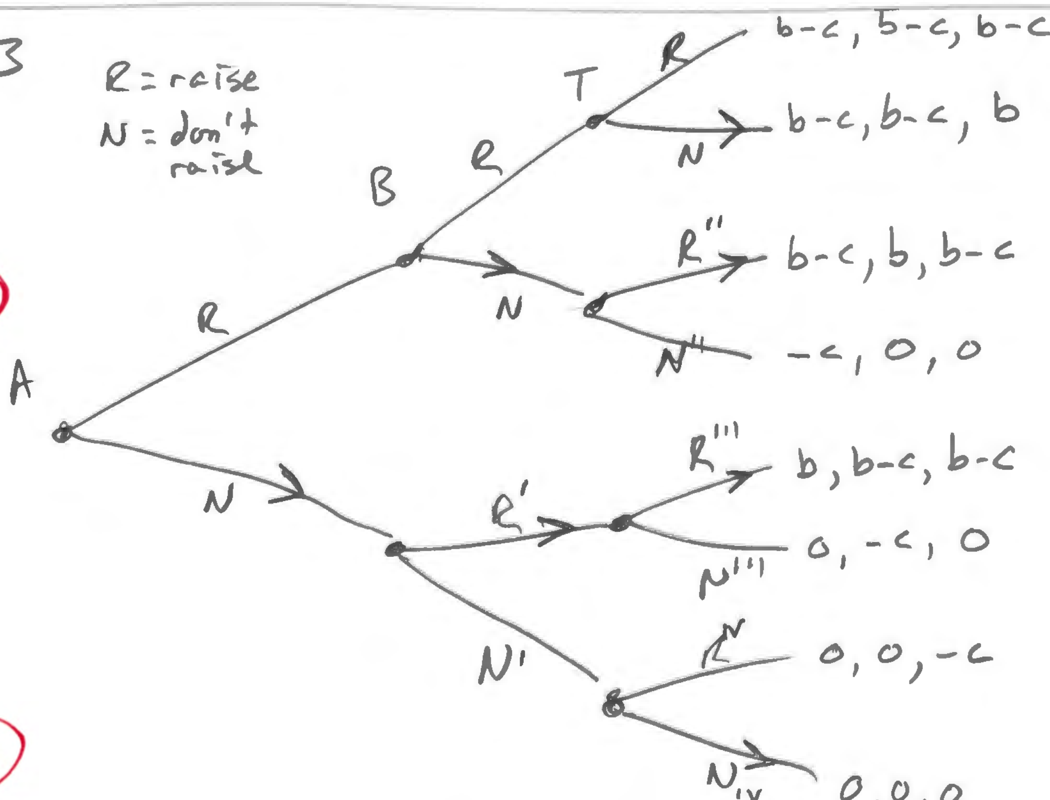


SPE:
P1 chooses Dℓ
P2 chooses D
payoffs (2,6)

#3

R = raise
N = don't raise

(a)



(b)
SPE:
P1 chooses N
P2 chooses NR'
P3 chooses $NR''R'''N^{IV}$
payoffs
(b, b-c, b-c)

(c)

Yes, by backward induction, you can vote "no" first, knowing the other two will vote "yes" and you avoid the "c" cost. Moving last, you know you will always have to vote "yes" because the other 2 players will always leave you the "yes" vote if they can.

#4

(a) $P = 300 - 2Q \rightarrow MR = 300 - 4Q$

Monopoly: $MR = MC_2 \rightarrow Q$

$$300 - 4Q = 100$$
$$2\cancel{4}Q = \cancel{2}00$$
$$\boxed{Q = 50} \rightarrow P = 300 - 2(50) \rightarrow \boxed{\$200 = P}$$

$$\pi^m = (P - MC)Q = (200 - 100)(50)$$
$$\boxed{\pi^m = \$5{,}000} = \pi_2^m \quad ; \quad \boxed{\pi_1 = 0}$$

Coop Profits

$$q_1 = 25 \rightarrow \pi_1^D = (200 - 80)25 - 10 = \boxed{\$2{,}990 = \pi_1^D}$$
$$q_2 = 25 \rightarrow \pi_2^D = \tfrac{1}{2}\pi^M = \boxed{\$2{,}500 = \pi_2^D}$$

Fight Profits $P_1^F < P_2^F = MC_2$ so, let's say $P_1^F = MC_2 - \$1.^{00}$

$$P_2^F = MC_2 = 100 \Rightarrow P_1^F = \$99$$

at $P = \$99$, Player 2 loses money so they drop out, $q_2 = 0$

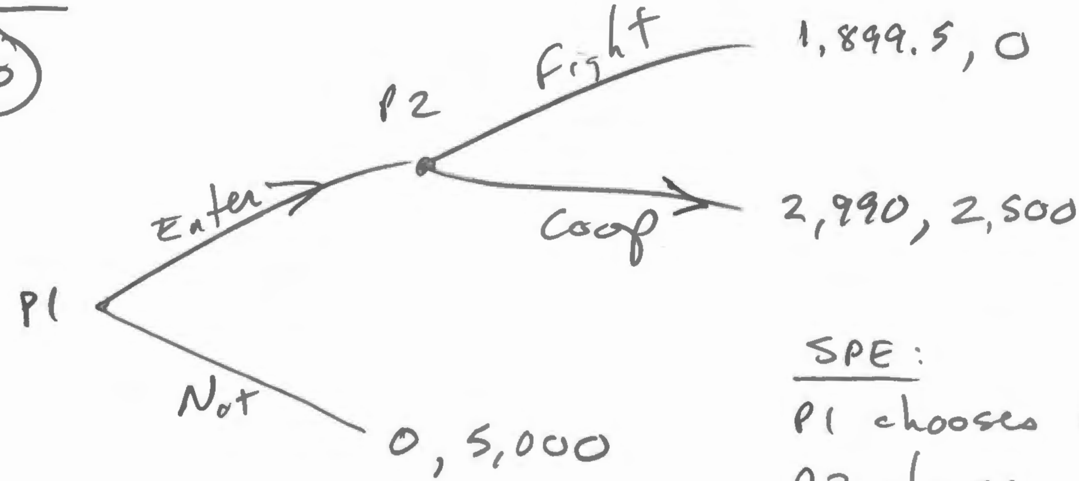
$$\boxed{\pi_2^F = 0}$$

To find q_1,
$$P = 300 - 2Q$$
$$99 = 300 - 2Q$$
$$2Q = 201$$
$$Q = \frac{201}{2} = \boxed{100.5 = Q}$$

$$\pi_1^F = (99 - 80)(100.5) - 10$$
$$\boxed{\pi_1^F = \$1{,}899.5}$$

#4

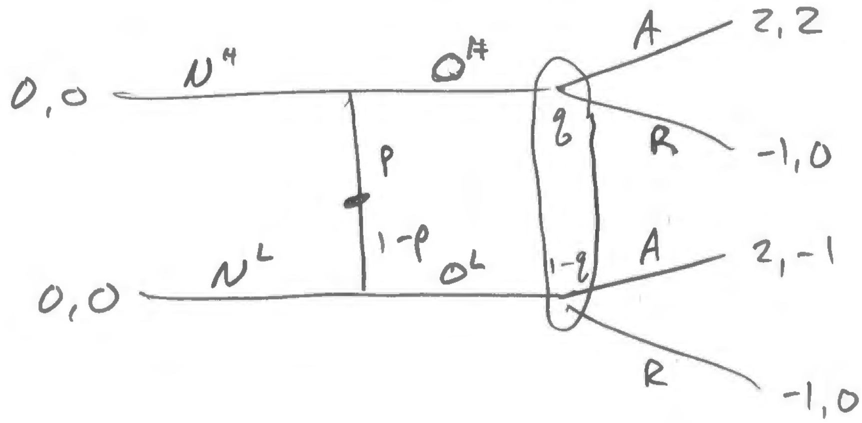
(b)



SPE:
P1 chooses Enter
P2 chooses Cooperate
$(\pi_1, \pi_2) = (2{,}900, 2{,}500)$

(c) The incumbent's threat to fight is not credible. This is an example of cheap talk. We discussed a number of ways for a player to gain credibility, however. For example, the incumbent could commit early to fighting by sinking some costs early (as we did in the Pepsi v.s. Coke game). Here it would still be difficult because P2's $mc > mc_1$. In the long-run, with this cost structure, P1 will likely drive P2 out of the market.
Perhaps P2 could invest early in lowering costs with the threat to commit to fighting.
Note: this is how competition drives technological development and efficiency.

#5



ⓐ Separating:

$N^H O^L$: $q=0 \Rightarrow$ P2 plays R $\Rightarrow$ P1 plays $N^L \Rightarrow$ Not EQm

$N^L O^H$: $q=1 \Rightarrow$ P2 plays A $\Rightarrow$ P1 plays $O^H O^L \Rightarrow$ Not EQm

ⓑ Pooling:

$O^H O^L$: $EU(A) > EU(R)$

$2p + (1-p)(-1) > 0$

$3p > 1 \Rightarrow \boxed{p > 1/3 \Rightarrow \text{play } A}$

If P2 plays A, P1 plays $O^H O^L \Rightarrow$ $\boxed{\text{This is an EQm}}$

if $p < 1/3$ P2 plays R & P1 plays $N^H N^L \Rightarrow$ not EQm

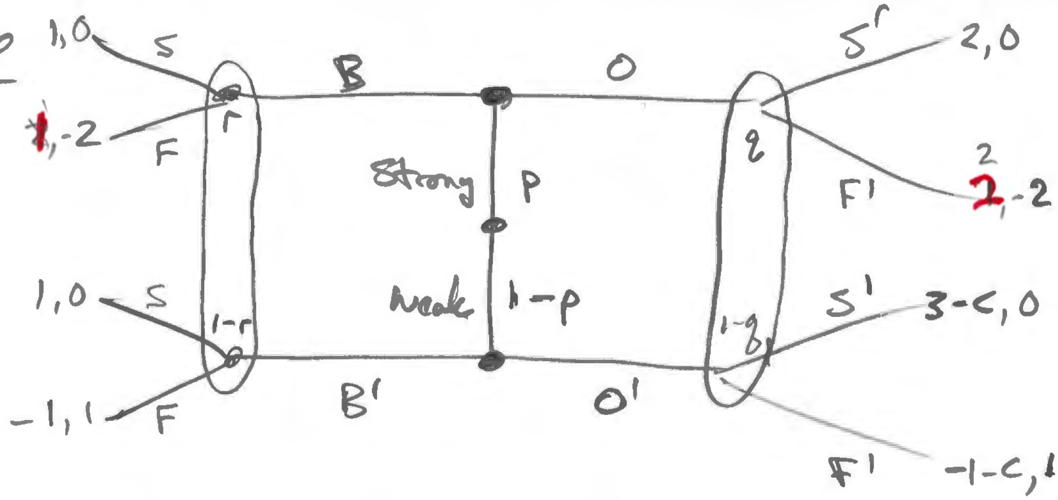
$N^H N^L$ as we saw above, if $p < 1/3$, P2 plays R and P1 will play $N^H N^L \Rightarrow$ This is an EQm

if $p > 1/3$, then P2 chooses A & P1 doesn't play $N^H N^L$, so this wouldn't be an EQm.

I accepted $N^H N^L$ is always a possible EQm ~~#~~ since P2 doesn't get a choice.
But you had to explain that.

Final Key 5

#6



P2's choices

play S' iff $EU(S') > EU(F')$

$0 \cdot q + 0(1-q) > -2q + 1-q$

$3q > 1 \rightarrow \boxed{\begin{array}{l} q > 1/3 \rightarrow \text{play } S' \\ q < 1/3 \rightarrow \text{play } F' \end{array}}$

play S iff $EU(S) > EU(F)$

$0 > -2r + 1-r$

$3r > 1 \rightarrow \boxed{\begin{array}{l} r > 1/3 \rightarrow \text{play } S \\ r < 1/3 \rightarrow \text{play } F \end{array}}$

(a) Separating

BO' $\left.\begin{array}{l} r=1 \\ q=0 \end{array}\right\}$ P2 plays SF' → P1 plays OB' if $\left.\begin{array}{l} 1 > -1-c \\ c > -2 \end{array}\right\}$ not EQM

and

P1 plays OO' if $c < -2$ } not EQM

So BO' is not a part of any separating EQM, no matter what the value of c.

p played no role in this.

Separating Con't

OB': $\left.\begin{array}{l} q=1 \\ r=0 \end{array}\right\}$ P2 plays S'F $\rightarrow$ P1 plays OB' if $\left.\begin{array}{l} -1>4-c \\ c>4 \end{array}\right]$ EQM

but P1 plays OO' if $c<4$ $\rightarrow$ not EQM

So, P1 plays OB' is an Equilibrium if $c>4$. P played no role here either.
P2 plays S'F

ⓑ Pooling

OO': $r=q=p$ P2 plays S'S if $p>1/3$ & F'F if $p<1/3$

S'S $\rightarrow$ P1 plays OO' if $\left.\begin{array}{l} 3-c>1 \\ c<2 \end{array}\right]$ EQM

and

P1 plays OB' if $c>2$] not EQM

F'F $\rightarrow$ P1 plays OO' if $\left.\begin{array}{l} -1-c>-1 \\ c<-2 \end{array}\right]$ EQM

and

P1 plays OB' if $c>-2$] not EQM

BB': $r=q=p$ P2 plays S'S if $p>1/3$ & F'F if $p<1/3$

S'S $\rightarrow$ P1 plays O on top, so BB' is not an EQM no matter what C is.

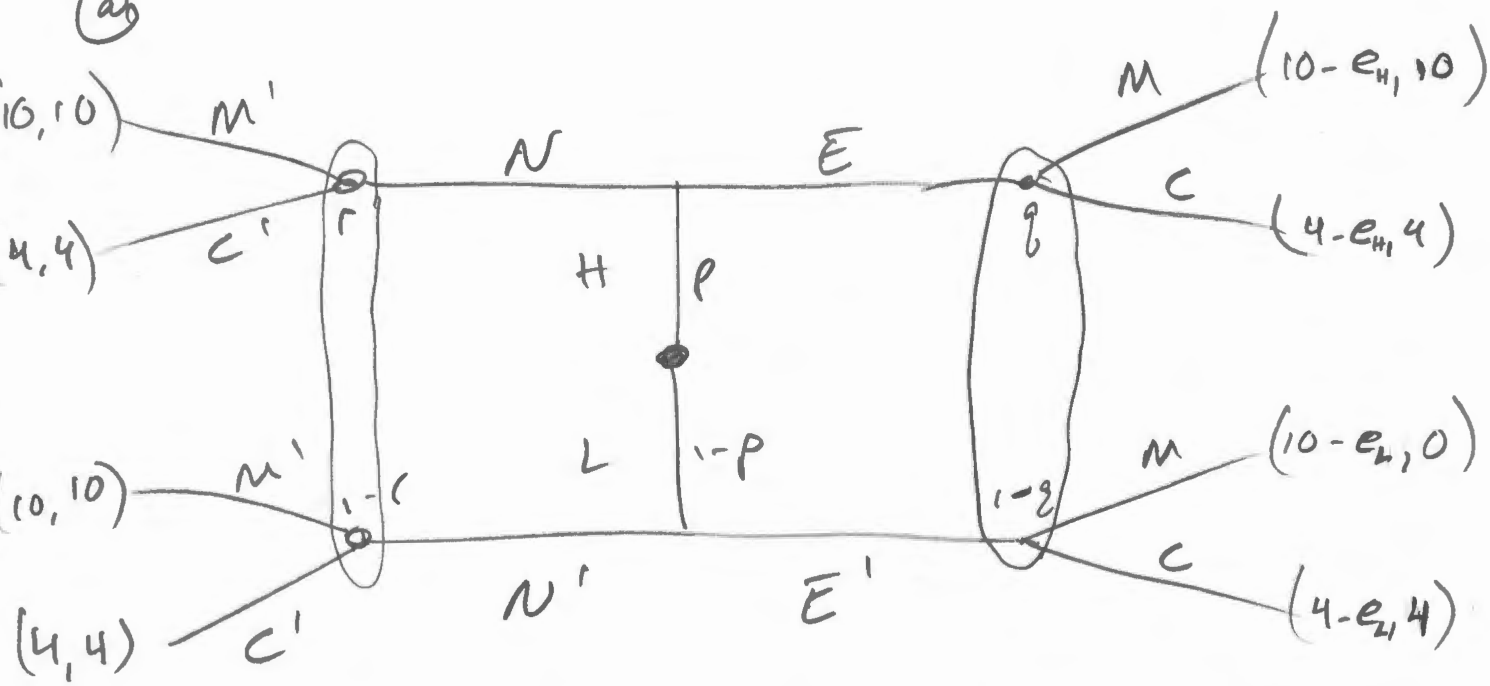
F'F $\rightarrow$ P1 plays O on top, so BB' isn't an EQM no matter what C is.

This says BB' is never an EQM choice, no matter what p or c are since O is always the dominant strategy for P1 at the top.

#7 Signalling Game

Final Key 7

(a)



(a) M if $EU(M) > E(C) \rightarrow 10p + 0(1-p) > 4p + (1-p)4$

$$\overset{5}{\cancel{10}}p > \overset{2}{\cancel{4}}$$

$$\boxed{p > 2/5}$$

Initially, in problem, we had $p = 1/3 < 2/5$, so employers chose C if they couldn't distinguish types.

If $p > 2/5$ and prob. of getting H type is high enough to risk putting everyone in M jobs.

(b) For H-types to choose E (in a separating eqm), we need $\underbrace{10 - e_H}_{\substack{P1:E \\ P2:M}} > \underbrace{4}_{\substack{P1:N \\ P2:C}} \Rightarrow \boxed{e_H < 6}$ So, if the costs of getting the education ever rise above 6, H-types also choose N. Our only eqm will be NN. This is because the payoff to E is too low. Given $p < 2/5$, employers don't risk M. So, NN', CC' is the EQM