Risk Analysis: Chap 13

pr (A) = chance A will occur

- when thinking about choices/decisions, the key is what people BELIEVE is the prob. This could be based on data or motion and usually is both:

  Ex: - gambling - house has clear statistical advantage - born out in data - but "I feel lucky"

  - sports
  - stock mkts

→ Subjective Probability

Prob Dist

Ex: Adept Tech Corp

	Pr
New Robot in 1 yr	.6
No New Robot in 1 yr	.4
	1.00

$\pi(\text{Robot}) = \$1{,}000{,}000$

$\pi(\text{No Robot}) = -\$600{,}000$

Expected π

$$E\pi = \sum P_i \pi_i = .6\,(1{,}000{,}000) + .4\,(-600{,}000) = \$360{,}000$$

Jones Corp: $\uparrow P_{tires}$ by $\$1.^{00}$
Tire Company

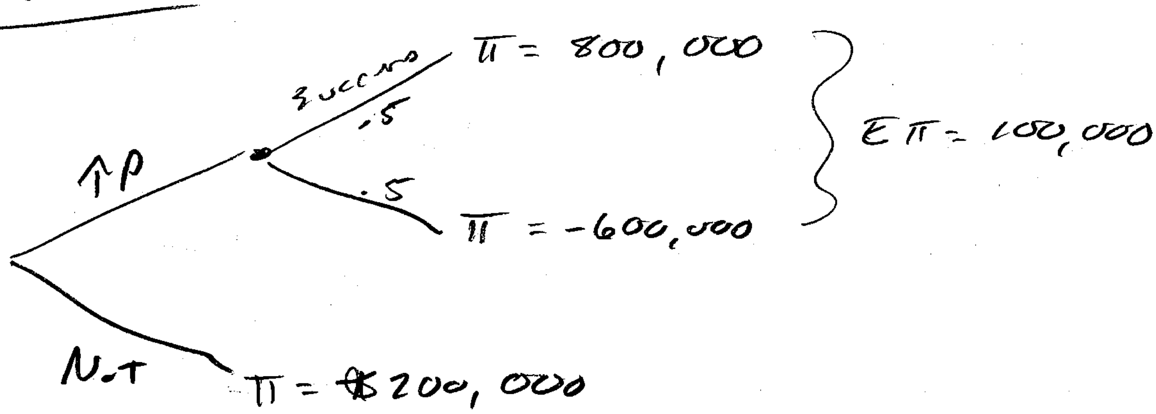
if Ad campaign successful $\rightarrow \pi^S = 800{,}000$ — Pr .5

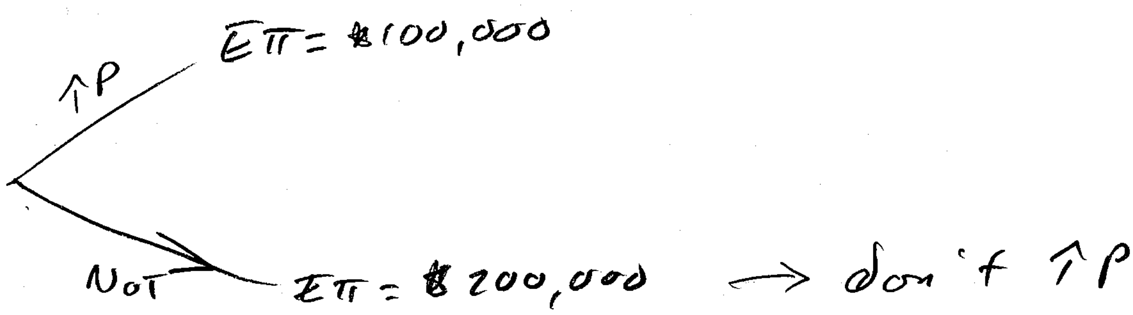
if NOT $\rightarrow \pi^N = -600{,}000$ — Pr .5

$$E\pi = .5(800{,}000) + .5(-600{,}000) = \$100{,}000$$

No campaign (ie $\Delta P = 0$) $\rightarrow \pi = \$200{,}000$

Decision Tree





Tomco Oil Corp – Decision To Drill

- clearly, don't know for sure if there is oil
- equipment ↑ accuracy but still not perfect

Decision: Drill, Not

If Drill & No Oil, $\pi = -\$90{,}000$

Drill & $q = 10{,}000$ barrels, $\pi = \$100{,}000$

Drill & $q = 20{,}000$, $\pi = \$300{,}000$

Drill & $q = 30{,}000$, $\pi = \$500{,}000$

If No Drill, $\pi = 0$

Looks potentially profitable, so hire geologist team

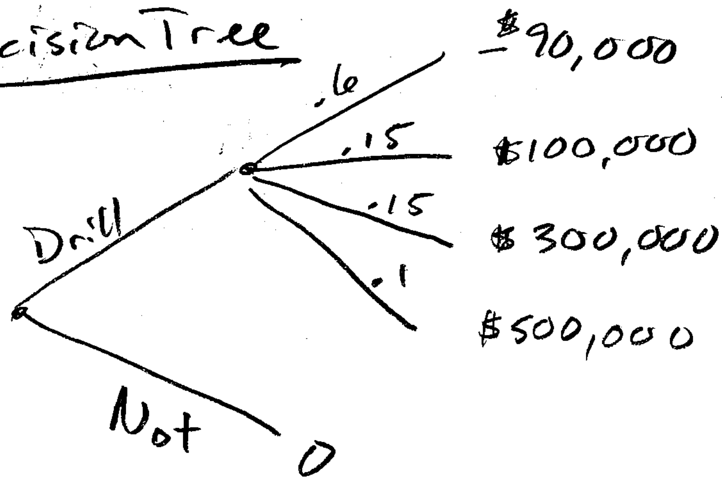
& learn $pr(\text{no Oil}) = .6$

$pr(q = 10{,}000) = .15$

$pr(q = 20{,}000) = .15$

$pr(q = 30{,}000) = .1$

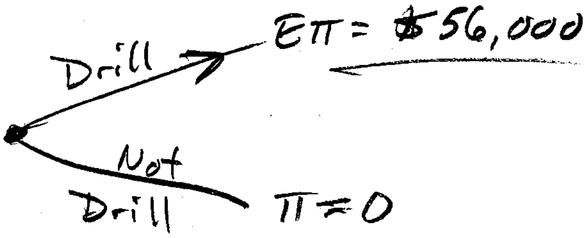
Decision Tree



Simplify Tree

$E\pi = .6(-90{,}000) + .15(100{,}000) + .15(300{,}000) + .1(500{,}000)$

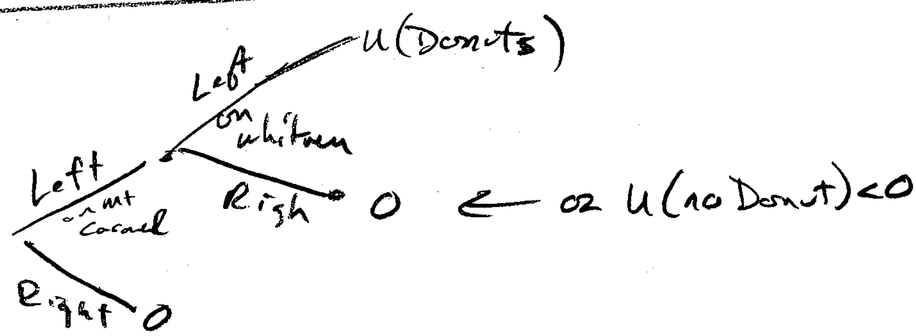
$\boxed{E\pi = \$56{,}000}$

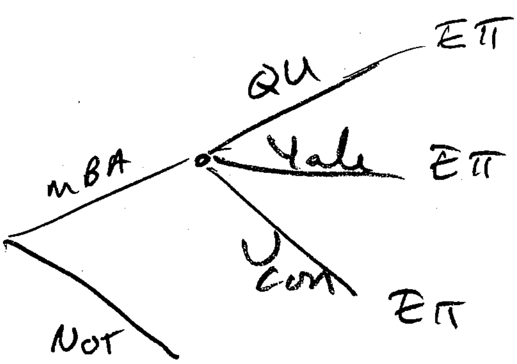


Notice: You solve such decision problems backwards from the way they unfold in reality

Tree Examples

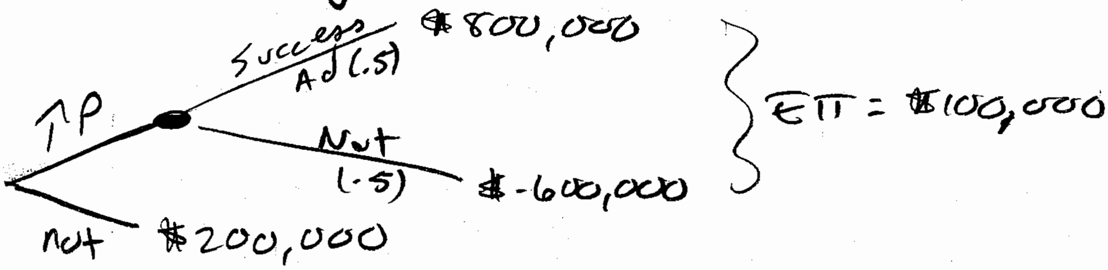
QU to Dunkin Donuts





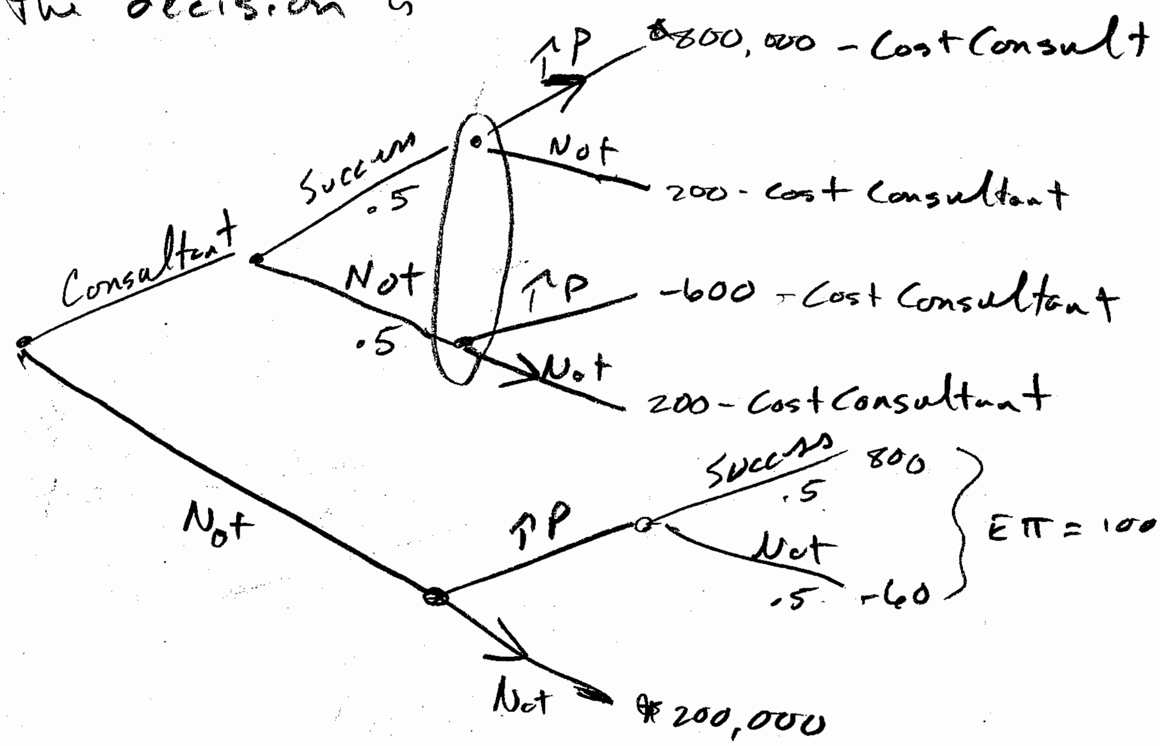
Value of Perfect Info

Jones Corp Again:



Imagine there is a perfectly accurate Ad Consultant who can tell Jones in advance whether the ad campaign will succeed or not.

Now the decision is



Note: Underlying prob(success) = .5 is unchanged. The only thing you purchased was WHEN you learn this.

"Info Cloud" helps you see where the uncertainty is in the decision tree.

Value of Perfect Info (Con't)

Do we hire the consultant?

$$E\pi_{consultant} = .5(800 - Cost) + .5(200 - Cost)$$

$$\boxed{E\pi_c = 500{,}000 - Cost}$$

$$\boxed{E\pi_{no\ Consult} = 200{,}000}$$

What's the max you would pay for the consultant?

To HIRE

$$E\pi_c = 500{,}000 - Cost \geq 200{,}000 = \pi_{not}$$

$$500{,}000 - 200{,}000 \geq Cost$$

Hire if $\boxed{Cost \leq \$300{,}000}$

If Cost > \$300,000, then $\pi_{NOT} > \pi_{consult}$ ∴ we shouldn't hire

So \$300,000 is the max consultant fee

or, it is the value to the Jones Corp of buying information.

Tomco Oil Again

Sppe Cost Drill = \$95,000

now $pr(\text{no Oil}) = .60 \Rightarrow \pi = -90 - 100 = -\$190{,}000$

$pr(10{,}000) = .15 \rightarrow \pi = 100 - 100 = \$0$

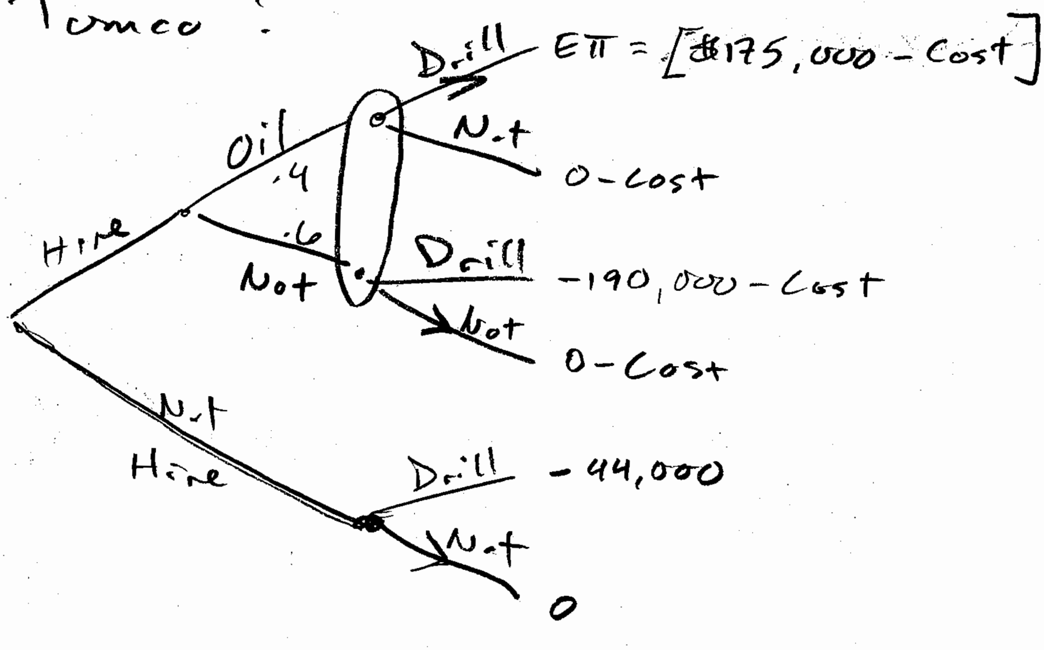
$pr(20{,}000) = .15 \Rightarrow \pi = 300 - 100 = \$200{,}000$

$pr(30{,}000) = .10 \Rightarrow \pi = 500 - 100 = \$400{,}000$

Now, $E\pi_{drill} = -\$44{,}000$ ∴ Choice is NOT Drill.

Geologists Develop new technology to tell you in advance

How much can the geologist team charge Tomco?



Show after we discuss updating probs.

we need to update the probs of # barrels if there is oil.

We used to have

$$\begin{aligned} pr(\text{No Oil}) &= .6 \\ pr(10k) &= .15 \\ pr(20k) &= .15 \\ pr(30k) &= .1 \end{aligned} \Bigg\} .4$$

So, it's clear $pr(\text{No Oil}) = .6$

$pr(\text{Oil}) = .4$

If there's oil, then what is prob of 10, 20, 30k ?

Conditional Prob = $pr(10k \mid \text{Oil}) = 15\%$ of 40% chance

$pr(20k \mid \text{Oil}) = 15\%$ of 40%

$pr(30k \mid \text{Oil}) = 10\%$ of 40%

So $pr(10k \mid \text{Oil}) = \frac{.15}{.40} = .375$

$pr(20k \mid \text{oil}) = \frac{.15}{.40} = .375$

$pr(30k \mid \text{oil}) = \frac{.1}{.4} = .25$

Check: $.375 + .375 + .25 = 1.00$ ✓ ok.

So

$E\pi_{drill} \mid \text{oil} = .375(0) + .375(200{,}000) + .25(400{,}000)$

$= \$175{,}000$ – Cost of study. Now Return to Tree

Tomco Oil Again (Con't 2)

Finally we need to compare Hire vs Not Hire and make our choice:

$$E\pi_{Hire} = .4\,(175\,000 - Cost) + .6\,(0 - Cost)$$
$$= .4(175{,}000) - Cost$$

$$\boxed{E\pi_{HIRE} = \$70{,}000 - Cost}$$

$$\boxed{E\pi_{not\,Hire} = 0}$$

Hire IF $\quad 70{,}000 - Cost \geq 0$

$$\boxed{Cost \leq \$70{,}000}$$

This is Value of Info and it is max that the Geology Firm can charge us.

Measuring attitude toward Risk

Coin Toss if H $\pi = \$1.^{00}$
if T $\pi = -\$1.^{00}$ } Game ①

what is $E\pi$? $pr(H) = .5 = p(T)$

$$\boxed{E\pi = .5(1) + .5(-1) = 0}$$

How much would you pay to play this?

Based on our past discussion ⇒ ZERO.

But people play this kind of game all the time.

How would you feel about this? 62

if H, $\pi = \$100$, if T, $\pi = -\$100$ → $E\pi = 0$

And if H, $\pi = \$100{,}000$, if T, $\pi = -\$100{,}000$ $E\pi = 0$ 63

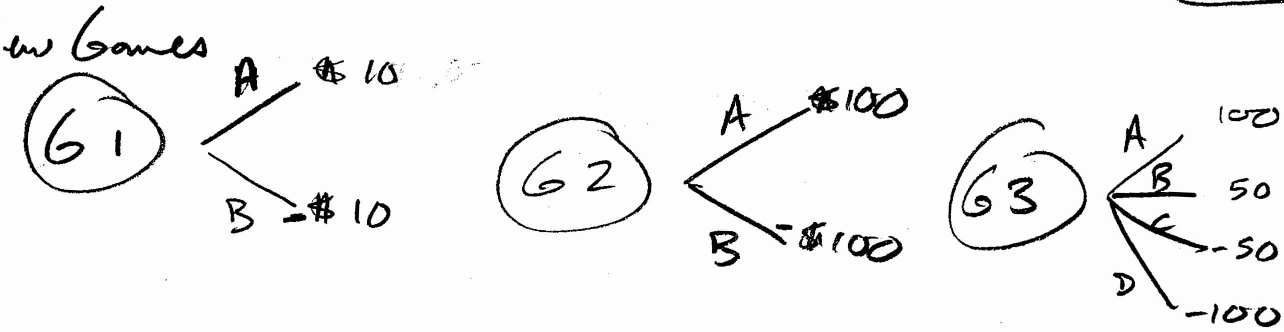
ARE they really the same?

which is more risky? 63 why?

How can we measure that?

Measuring Risk

New Games



Range 1 Max - Min = 20

Range 2 Max - Min = 200 ⟵ More Risk!

But

Range 3 Max - Min = 200 Looks Same.

Problem w range is we didn't use ALL info.

Also we need a basis for comparison, reference base. Use mean.

Measure 2 for G1 $10-\mu + (-10-\mu) = 0$

$100-\mu + (-100-\mu) = 0$

Square Distance from Mean

for G1 $(10-\mu)^2 + (-10-\mu)^2 = (10)^2 + (-10)^2 = 200$

G2 $(100-\mu)^2 + (-100-\mu)^2 = 100^2 + (-100)^2 = 20{,}000$

$10{,}000 + 10{,}000$

G3 $(100)^2 + 50^2 + (-50)^2 + (-100)^2 = 25{,}000$ Diff

$10000 \quad 2500 \quad 2500 \quad 10{,}000$

Normalize $\frac{200}{2} = 100$, $\frac{20{,}000}{2} = 10{,}000$, $\frac{25{,}000}{4} = 6{,}250$

Least RISKIEST middle

The measure we developed is called

VARIANCE, $\sigma^2 = \frac{\sum (x-\mu_x)^2}{N}$

OR

$$\sigma^2 = \sum Pr_i (x_i - \mu_x)^2$$

SAN JOSE

$pr(\text{New Robot}) = .6 \rightarrow \pi = 1{,}000{,}000$

$pr(\text{None}) = .4 \qquad \pi = -600{,}000$

$\mu = E\pi = \sum Pr_i \pi_i = .6(1{,}000{,}000) + .4(-600{,}000) = \$360{,}000$

$Var(\pi) = \sum Pr_i (\pi_i - \mu)^2 = .6(1{,}000{,}000 - 360{,}000)^2 + .4(-600{,}000 - 360{,}000)^2$

$= 614{,}400{,}000{,}000$

JONES

$\mu = E\pi = .5(800) + .5(-600) = 100$

$Var(\pi) = .5(800-100)^2 + .5(-600-100)^2 = 490{,}000$

Problem is that var = $\$^2$ so to get rid of square,

Std Dev (San Jose) = \$783,836

Std Dev (Jones) = \$700 k

Std Dev is better cause it's in \$ terms. Still tough to compare for decisions. So...

Coefficient of Variation

$$V = \frac{\sigma}{E(\pi)}$$

StdDev assumes that the scale of the projects is constant. $\uparrow \pi$ amt will always mean $\uparrow \sigma$

CoV scales by π so you consider "variation relative to π."

THESE are all just mathematical calculations. For decision analysis we need to know how the decision maker views or "feels about" risk.

Someone who is "RISK Neutral" has no special feeling about risk. They view it mathematically.

A "risk LOVER" may actually prefer the riskier situations

Someone who is "RISK AVERSE" weighs risk a lot.

People can be different in different contexts.

RISK AVERSE as CPA but
risk lover skydiving on weekend.

Utility, Feeling, and Risk

Constructing Utility: Pepsi, Coke, Water

$u(\text{Water}) > u(\text{Coke}) > u(\text{Pepsi})$

Pick easy numbers → 1 .5 0

If you have some info, use that. Sppe you see me buying cokes for \$1.00. Then you would know I'd pay more than \$1.00 for water but less for Pepsi

Getting Exact Values: Tomco Oil Again

$$E\pi = .6\underbrace{u(-90)} + .15u(100) + .15u(300) + .1\underbrace{u(500)}$$

Pin Down Extremes First.

Pick Easy Numbers $u(-90) = 0$
$u(50) = 50$

How to find $u(100)$ & $u(300)$?

Ask. What p leaves you indiff between

$$\underset{\text{for sure}}{u(100)} = pu(500) + (1-p)u(-90)$$

Sppe, after enough questioning, $p = .4$

$$u(100) = .4\underbrace{u(500)}_{50} + .6\underbrace{u(-90)}_{0}$$

$$\boxed{u(100) = 20}$$

Now do the same for $u(300)$

$$u(300) = p\,u(500) + (1-p)\,u(-90)$$

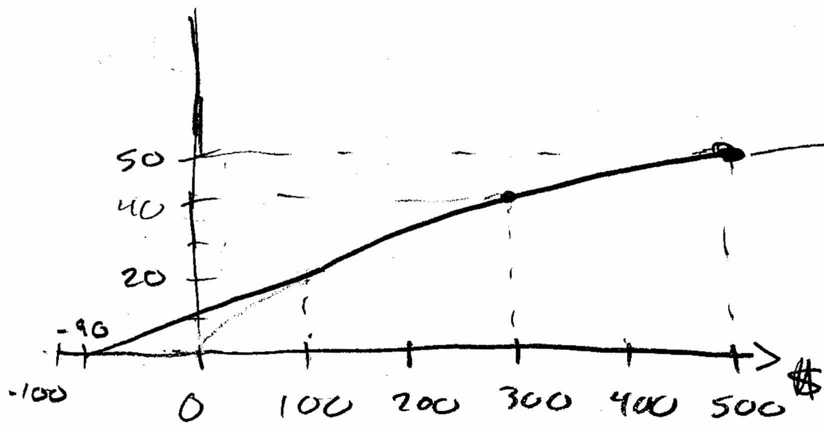
Again, after enough questioning (or experience), $P = .8$

$$\boxed{u(300) = .8(50) = 40}$$

must compare Eu vs u

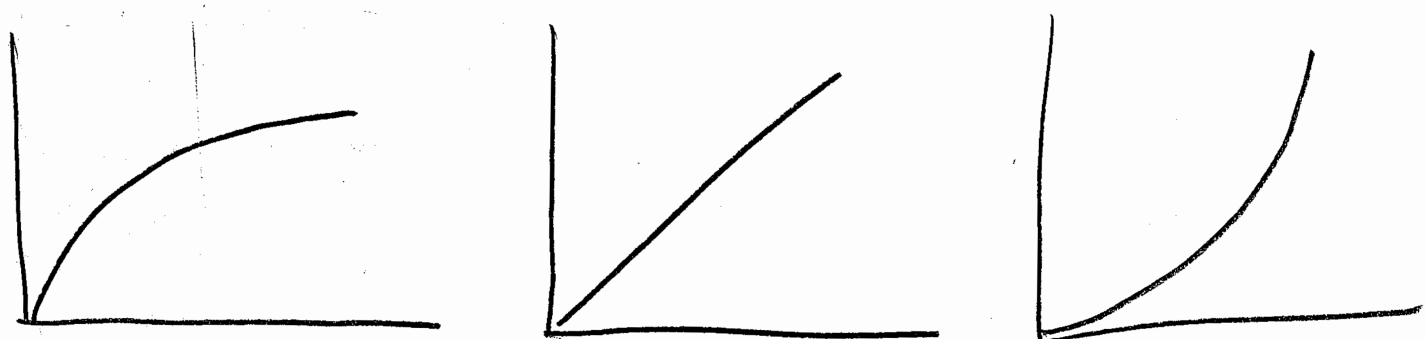
$$Eu = .6(0) + .15(20) + .15(40) + .1(50) = 14 \quad \text{vs } u(0) = 10$$

$\Rightarrow$ DRILL



Looks like prod. fn.

Turns out $\sqrt{\ }$ works well for risk Averse



Risk Averse

$Eu = \sum pr_i \sqrt{\pi_i}$

Risk Neutral

$Eu = a + bE\pi$

Line

Risk Lover

$Eu = \sum pr_i \pi_i^2$

Using Certainty Equivalence in Insurance

Manager holds mortgage debt = \$900 mill

$pr(\text{mkt worsen}) = .25 \rightarrow \downarrow$ value to \$400 mill

$pr(\text{no } \Delta) = .75$

$E\pi = .25(400) + .75(900) = \$775$ mill

Expected Value (also EU for Risk Neutral)

Risk Averse manager has $U = \sqrt{W}$

$$\boxed{EU = .25\sqrt{400} + .75\sqrt{900} = 27.5}$$

$$U = \sqrt{W} = 27.5 \Rightarrow \boxed{W = \$756.25 \text{ mill}}$$

This is the reservation price for managers.
If value $\downarrow$ below \$756.25, sell.

LBI Insurance Example: Insuring above manager.

LBI policy: full coverage (ie \$500 mill) $900 - 400 = 500$

LBI is Risk Neutral. what premium should it charge?

$E(\text{payout}) = .25(500) + .75(0) = \$125$

So, the policy must pay $\geq \$125$

$900 - 143.75$

Sipe: \$143.75 $\Rightarrow$ Managers have \$756.25 for sure

Since $u(756.25) = EU(\text{above})$, managers will buy.

Risk Premium for LBI $= \underset{\text{price}}{\$143.75} - \underset{\text{payout}}{\$125} = \$18.75$

If the risk premium is between \$125 & \$143.75, then managers prefer the insurance to the risk because $u(900 - \text{Premium}) > u(\text{Risk}) = 27.5$

And the insurance company will sell the policy because $E\pi$ of Premium $- 125 > 0$

If LBI has enough market power, it will charge as close to \$143.75 as possible.

Putting It All Together: Adverse Selection in Insurance Market

Insurance Company Wants to offer insurance to drivers

Consider 2 drivers. Both with wealth, $W_0 = 125$.
Crash $= -100$ Loss

2 Types:
- High Risk $\rightarrow pr_{HR}(\text{Crash}) = .75$
- Low Risk $\rightarrow pr_{LR}(\text{Crash}) = .25$

So,

$E\,Loss_{HR} = pr(Loss)\,Loss = .75(-100) = -75$

$E\,Loss_{LR} = pr(Loss)\,Loss = .25(-100) = -25$

In a perfectly competitive market with Full and Perfect Information, HR people would pay 75 for their insurance an LR people would pay 25 for theirs.

Call these $P_{HR} = 75$

& $P_{LR} = 25$

Perfect Information : Assume Risk Aversion

$u = \sqrt{W}$

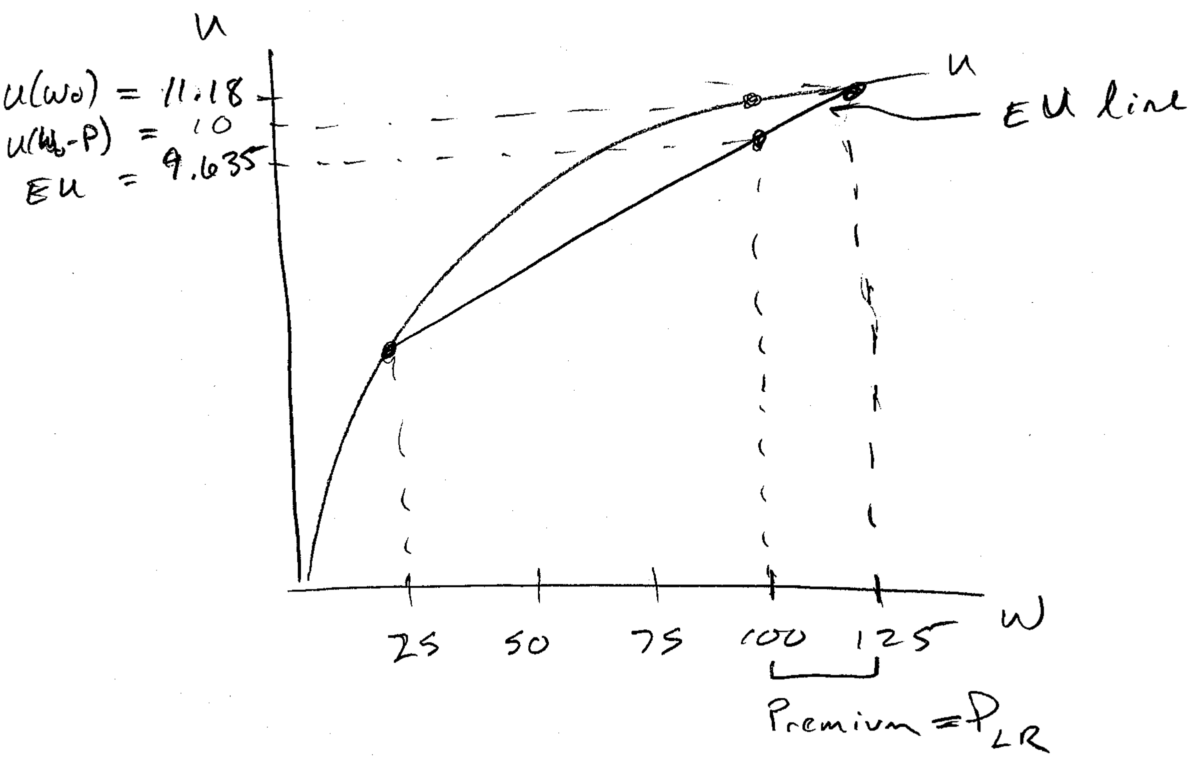
Low Risk $\quad u(\text{with insurance}) = u(W_0 - P_{LR}) = \sqrt{100 - 25} = 10$

$$EU(\text{no insurance}) = pr(\text{No Crash})\,u(W_0) + pr(\text{Crash})\,u(W_0 - P_{HR})$$

$$= .75\sqrt{125} + .25\sqrt{100-75}$$

$$= .75\sqrt{125} + .25\sqrt{25}$$

$$EU_{LR}(\text{no Ins}) = 9.635$$

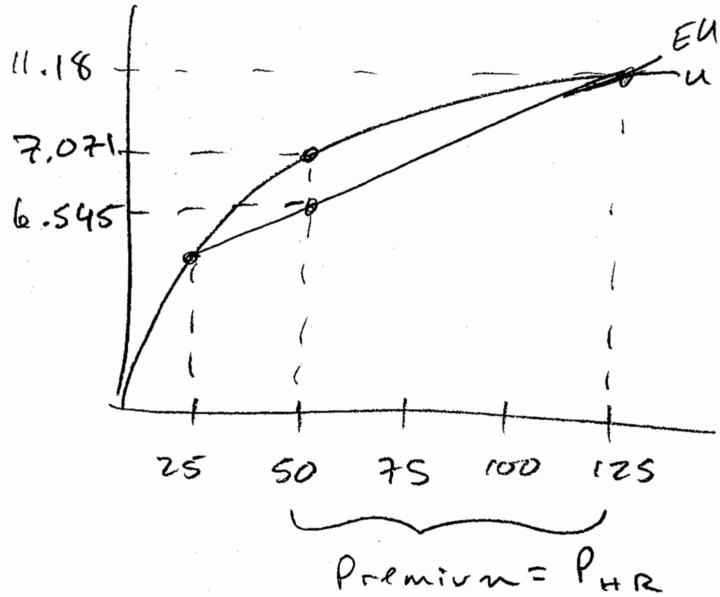


High Risk

$75 - P_{HR}$

$u(\text{w/ ins}) = \sqrt{125-75} = 7.071$

$u(\text{no ins}) = .25\sqrt{125} + .75\sqrt{25} = 6.545$



So both are better off with insurance than without.

That is, the LR person prefers 100 with certainty to EU and the HR person prefers 50 w/ certainty to EU

Imperfect Information

Suppose Insurance company can't tell them apart.

They know that, in the population overall, about half are HR & half are LR.

The company can break even by charging a premium:

$$.5(25+75) = 50 = \bar{P}$$

But who will buy?

High Risk:

$$U(\text{Insurance}) = \sqrt{W_0 - \bar{P}} = \sqrt{125-50} = 8.66$$

So $$U(\text{Insurance}) = 8.66 > EU(\text{no Insurance}) = 6.545$$

So, high risk buys.

Low Risk

$$U(\text{Insurance}) = \sqrt{W_0 - \bar{P}} = \sqrt{125-50} = 8.66$$

$$U(\text{Insurance}) = 8.66 < EU(\text{no Ins}) = 9.435$$

$\rightarrow$ Do not buy.

So only High Risk buy...