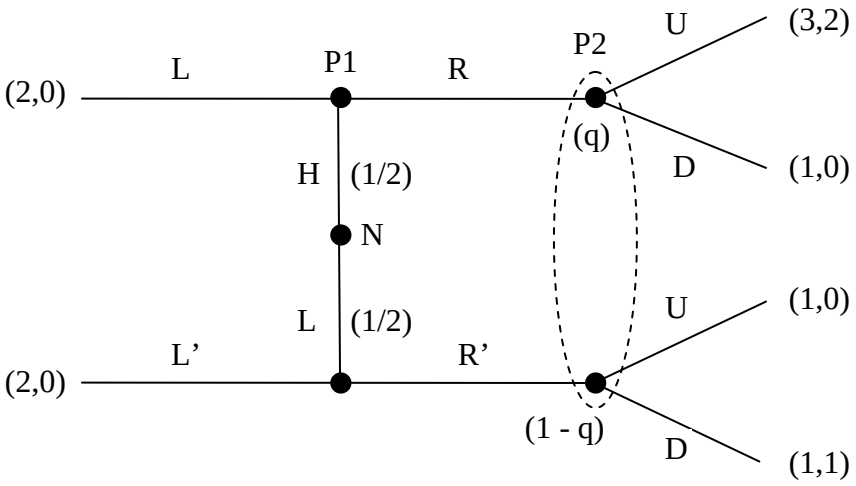
Asymmetric Information Games I

1. Pre-packaged Asymmetric Game



a. Does this game have a *separating* perfect Bayesian equilibrium? If so, fully describe it.

b. Does this game have a *pooling* perfect Bayesian equilibrium? If so, fully describe it.

2. Entry Game (Watson, Chap. 24)

Consider the following market game. There are two firms: the "incumbent" and the "entrant". The incumbent firm has either high costs (H) or low costs (L); this is the incumbent's type, which is selected by nature at the beginning of the game. With probability q, the incumbent's type is H and, with probability 1 – q, the incumbent's type is L. The incumbent observes its own type, but the entrant does not observe the incumbent's type. After observing its type, the incumbent selects either a high price (pH) or a low price (pL). The entrant observes the incumbent's price and then decides whether or not to enter the market (E or N). The incumbent's payoff is 0 if the entrant chooses E (regardless of the incumbent's type). If the entrant picks N and the incumbent's price is pH, then the high-type incumbent gets 2 and the low-type incumbent gets 4. If the entrant picks N and the incumbent's price is pL, then the high-type incumbent gets 0 and the low-type incumbent gets 2. The entrant obtains nothing if it does not enter. It obtains a payoff of 1 if it enters and faces the high-type incumbent. It obtains -1 if it enters and faces the low-type incumbent.

a. Draw the game tree.

b. Find a separating perfect Bayesian equilibrium for this game.

c. Find a pooling perfect Bayesian equilibrium for this game. Under what values of q does it exist?

3. Mall Game (Watson, Chap. 28)

Consider a game between two friends, Amy and Brenda. Amy wants Brenda to give her a ride to the mall. Brenda has no interest in going to the mall unless her favorite shoes are on sale at the large department store there. Amy likes these shoes as well, but she wants to go to the mall even if the shoes are not on sale. Only Amy subscribes to the newspaper, which carries a daily advertisement of the department store. The advertisement lists all items that are on sale, so Amy learns whether or not the shoes are on sale. Amy can prove whether or not the shoes are on sale by showing the newspaper to Brenda. But this is costly for Amy, because she will have to take the newspaper away from her sister, who will yell at her later for doing so.

In this game, nature first decides whether or not the shoes are on sale, and this information is made known to Amy. (Amy observes whether nature chose S or N.) Nature chooses S with probability p and N with probability 1 – p. Then Amy decides whether or not to take the newspaper to Brenda (T or D). If she takes the newspaper to Brenda, then it reveals to Brenda whether the shoes are on sale. In any case, Brenda must decide whether to take Amy to the mall (Y) or to forget it (F). If the shoes are on sale, then going to the mall is worth 1 unit of utility to Brenda and 3 to Amy. If the shoes are not on sale, then traveling to the mall is worth 1 to Amy and -1 to Brenda. Both players obtain 0 utility when they do not go to the mall. Amy's personal cost of taking the newspaper to Brenda is 2 units of utility, which is subtracted from her other utility amounts.

a. Draw the game tree.

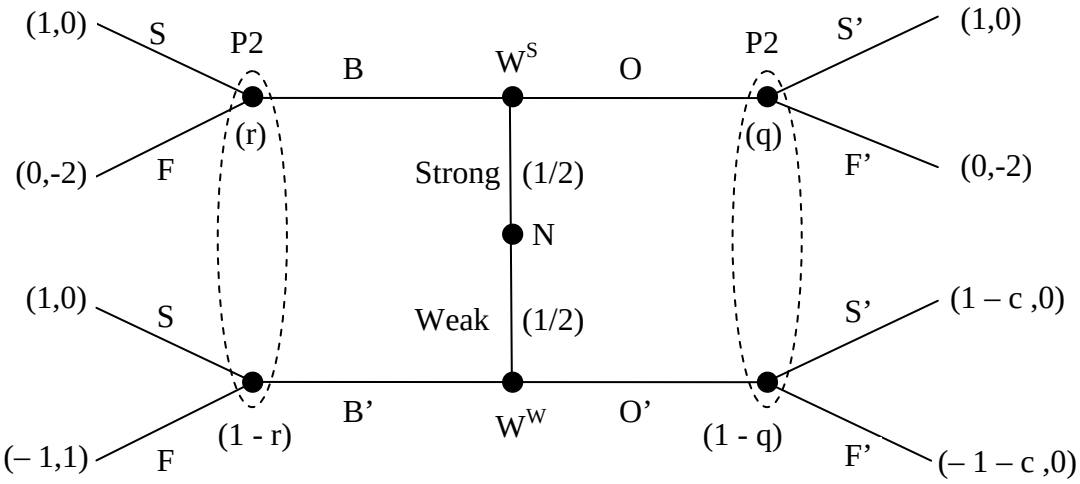
b. Does this game have a separating perfect Bayesian equilibrium? If so, fully describe it.

c. Does this game have a pooling perfect Bayesian equilibrium? If so, fully describe it.

4. The Princess Bride Game

In the classic Rob Reiner movie The Princess Bride, there is a scene at the end where Wesley (the protagonist) confronts the evil prince Humperdinck. The interaction can be modeled as the following game. Wesley is one of two types: weak or strong. Wesley knows whether he is weak or strong, but the prince only knows that he is weak with probability ½ and strong with probability ½. Wesley is lying in bed in the prince's castle when the prince enters the room. Wesley decides whether to get out of bed (O) or stay in bed (B). The prince observes Wesley's action but does not observe Wesley's type. The prince then decides whether to fight (F) or surrender (S) to Wesley. The payoffs are such that the prince prefers to fight only with the weak Wesley, because

otherwise the prince is an inferior swordsman. Also, the weak Wesley must pay a cost c to get out of bed. The extensive-form representation of the game is:



a. What conditions on *c* guarantee the existence of a separating PBE? Fully describe such an equilibrium.

b. For what values of *c* is there a pooling equilibrium in which both strong and weak Wesley get out of bed? Fully describe such an equilibrium.