Debt maturity choices when capital inflows can suddenly stop.

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Much of the literature on capital inflow sudden stops uses models with short term debt only but draws conclusions about the role of debt maturities in crises. In this paper short and long term debt are introduced in an infinite horizon, representative agent model that is consistent with modern open economy macroeconomic models of the type presented in Obstfeld and Rogoff (1996). Phase diagrams are derived for the paths of consumption and debt in an economy suffering from sudden stops. Short-term capital controls are discussed as an application of the model to the policy-relevant debates.

JEL Classification: F32 – Current Account Adjustment, Short-term Capital Movements, F34 – International Lending and Debt Problems, F41 – Open Economy Macroeconomics.

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1. Introduction

A sudden stop is said to occur when the capital flowing into an economy suddenly stops (Calvo 1998). Much of the sudden stop literature focuses on the causes of sudden

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stops, their monetary effects, resultant financial crises and relative price shocks. Many have concluded that an over reliance on short-term borrowing is partially to blame for sudden-stop drive crises. Consequently, a common policy recommendation is that there is a need to encourage longer maturity structures, perhaps though capital controls. Most research, however, models economies with short term borrowing only or that include different maturities in a Diamond-Dybvig (1983) setup where individuals aren't really smoothing per-date lifetime consumption.

This paper presents a representative agent model that includes both short and long term borrowing in an infinite horizon framework. Doing so allows one to ask steady state questions about the optimal maturity structure of debt when capital inflows can suddenly stop. An anticipated and an unanticipated sudden stop are considered as well as the effects of a policy forcing individuals to borrow long-term only.

The paper proceeds as follows. Section two presents a brief literature review and overview of the model. Section three presents the model. Section four discusses the model dynamics and phase diagrams. Section five explores the above mentioned sudden stop experiments in this model. Section six discusses the option of banning short term capital flows. Section seven concludes.

2. Overview and Literature Review

The first analytical attempt to understand sudden stops was in Calvo (1998). There he used simple macroeconomic accounting identities to illustrate the basic mechanism of an economic crisis coming through a country's capital account. Building on these insights, Calvo and Reinhart (2000) show that while sudden stop episodes empirically resemble standard balance of payments crises, they can result in longer and more pronounced recessions than typically observed. Calvo, Izquierdo and Talvi (2002) focus on the real exchange rate effects of a sudden stop in Argentina, again combining simple analytics with empirical data. Edwards (2004a) and Edwards (2004b) look at a thirty year history of sudden stops and the relationship between sudden stops and the degree of financial openness. Now there is even some popular press discussion as to whether the United States is subject to a sudden stop (Ip, 2005). That sudden stops are a problem thus seems fairly well documented at this point. Their exact nature, causes and consequences are still a topic of some debate, however.

While Calvo (1998) was the first analytical attempt at understanding sudden stops, it was by no means a model of the phenomenon. It wasn't until 2003 that Calvo (2003)[2] presented a more developed model of a sudden stop where growth is a negative function of an economy's fiscal burden. This is combined with the ability of growth to discontinuously switch from high to low (associated with a sudden stop) when the fiscal burden reaches a critical point, argues that sudden stop crises could be the result of fiscal distortions. Mendoza builds on his earlier model (Mendoza, 1991) of open economy business cycles in a series of three papers. Mendoza (2002) argues that sudden stops can be the outcome of a flexible-price economy with imperfect credit markets. Mendoza and Arellano (2002) review the varieties of capital market crisis models in the literature in the context of a small open economy real-business-cycle model where borrowing constraints occasionally bind. Mendoza and Smith (2002) and Mendoza and Smith (2004) propose an open economy asset pricing model with financial frictions that generates predictions in line with empirical observations. The sudden stop is driven in their model by shocks that

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[2] Actually the paper had already been presented at an IMF conference in 2002.

suddenly trigger international margin calls on domestic agents. A final approach worth attention is Caballero and Krishnamurty's (1998, 2000, and 2001). They view sudden stops as an underinsurance problem and search for optimal mechanisms that could be implemented as a solution.

While this body of research represents a major advance in our understanding of sudden stops and their associated problems, these models all assume short-term – per date or per instant – borrowing. Nevertheless, many (Calvo is one notable exception) conclude that a country's over-reliance on short term borrowing is problematic. When a sudden stop strikes, a country is forced to repay all its outstanding obligations. Longer-term maturities allow countries to spread this burden out. One natural policy prescription would then be to encourage longer maturity structures through capital controls.

An exception to the short-term borrowing only models has been the work of Chang and Velasco. Based on their earlier work (Change and Velasco 1998a and 1998b), Chang and Velasco (2000) as well as others (Rodrik and Velasco, 1999) have addressed sudden stops as a form of financial crisis in a Diamond-Dybvig (1983) type world. That is a three date world where individuals receive information about their endowment at the first date, but only consume at one of the last two dates. This allows one to handle maturity choices since individuals borrow (or deposit money at banks, depending on the setup) either short-term (for the second date consumers) or long-term (for third date consumers). Some uncertainty is introduced and crises can occur when coordination fails. This approach has added much to our understanding of the fragility of financial systems and why they are prone to collapse.

Despite the appeal of these three date models, they do not capture maturity choice in an intertemporal optimization framework, so much is missed. When individuals only consume at one date out of three possible, there is no intertemporal consumption to consider. As a result, a broader understanding of the role of maturities in complete infinite horizon models isn't possible. The results of this approach, while important, are very limited as well (Garber, 1999).

The present paper includes maturity choices in an infinite horizon model that allows intertemporal optimization in the spirit of Obstfeld and Rogoff (1996). This is a first step in examining the role of maturity choices in richer modeling environments. The model is of a cashless society represented by a single individual. Individuals receive a constant endowment and can borrow from the rest of the world. They have an incentive to do so based on a difference between their subjective rate of time preference and the riskless world interest rate. The interest rate charged them to borrow increases with the amount borrowed[3] and is based on the amount to be repaid at a given date. One explanation could be that lenders could consider the risk of default (not explicitly modeled) at a given date as rising with the stock of debt due at that date. An additional assumption is that lenders charge risk premia based on the aggregate, not the individual, level of borrowing. This is intended to capture the problem faced by many in emerging markets that they are charged a higher interest rate simply because the country in which they reside is considered risky. While this assumption has no influence on the relative price of different maturing debt and thus individual maturity choices, it does cause individuals to over borrow due to the divergence of private and social marginal costs.

[3] This approach follows Agenor (1997), Agenor and Montiel (1999), Auernheimer and Garcia-Saltos (2000), and Pitchford (1989 and 1991).

The main reasons for the assumption are that it captures a real world issue faced by emerging market borrowers and that it greatly adds to the model's tractability.

Sudden stops are modeled as discrete increases in the interest rate charged on new borrowing. Future work on sudden stops should explore three different types of uncertainty in this environment: uncertain magnitude (a sudden stop need not be so bad that all borrowing ceases), uncertain initial sudden stop date, and, uncertain duration. This paper represents an initial step. It models sudden stops as permanent and large enough that individuals choose to borrow nothing during the stop (i.e., are forced into autarky).

Finally, the effects of capital controls to encourage long-term borrowing are investigated by examining the extreme case of banning short-term borrowing. In this model such capital controls are unambiguously welfare decreasing. This is due to two features of the model. First, the steady state with only one maturity type will also exhibit higher borrowing costs, lower consumption and lower utility. Second, when sudden stops are anticipated individuals under capital controls are less able to smooth their consumption around the sudden stop date.

3. The Model

Individuals are assumed to have constant endowment, Y_t, which they know with certainty. Lenders do not know the economy's endowment with certainty and charge risk premia. Borrowers are small in world capital markets and there is zero private verification of borrowing. As a result, individually they view the interest rate charged them for international funds as given. Collectively, however, the amount of aggregate debt due at a given date determines the interest rate charged on borrowing. The result is over borrowing due to the difference between the private and social marginal cost of borrowing while the choice between maturities remains unaffected. The model has no money and only one good which is traded. This allows one to focus clearly and exclusively on the role of debt and its maturity.

Individuals are modeled as a single representative individual. Individuals maximize the following lifetime utility functional which is time separable

$$(1) \qquad U = \sum_{s=t}^{\infty} \beta^{s-t} u(C_s) \qquad with \quad \beta \in (0,1]$$

subject to the flow budget constraint

$$(2) \quad Y_t + (1+r^w)\,{}_{t-1}S_t + {}_tB_{t+1} + {}_tB_{t+2} = C_t + (1+{}_{t-1}r_t)\,{}_{t-1}B_t + (1+{}_{t-2}r_t)^2\,{}_{t-2}B_t + {}_tS_{t+1}$$

where β is a constant subjective discount factor and C_t is date t consumption. All other variables are determined at the pre-subscripted dates and predetermined at their post-subscripted dates. Accordingly, ${}_{t-1}S_t$ is short term – which will always mean 1-date – saving from the previous date, *t-1*, and returning the world real interest rate, r^w , upon maturity at date t. ${}_tB_{t+z}$ (or ${}_{t-z}B_t$) represents borrowing and comes in two different maturities. Short term borrowing, $z = 1$, charging a short-term interest rate and long term, $z = 2$, charging a long term interest rate squared. Without exception, *S* will be weakly positive and *B* is weakly positive but represents a liability. Thus, interest plus principle on savings appears with new short and long term borrowing as well as the endowment on the left hand side of the budget constraint while consumption, repayment of short and long term borrowing plus interest, and new savings appear together on the right hand side.

Taking the interest rates as given, the optimality conditions for the individual's problem (obtained by differentiating with respect to ${}_tS_{t+1}$, ${}_tB_{t+1}$, ${}_{t+1}B_{t+2}$, and ${}_tB_{t+2}$ respectively) are as follows.

(3) $$\frac{\partial U(C_t)}{\partial C_t} = \beta(1+r^w)\frac{\partial U(C_{t+1})}{\partial C_{t+1}}$$

(4) $$\frac{\partial U(C_t)}{\partial C_t} = \beta(1+{}_tr_{t+1})\frac{\partial U(C_{t+1})}{\partial C_{t+1}}$$

(5) $$\frac{\partial U(C_t)}{\partial C_t} = \beta^2(1+{}_tr_{t+2})^2\frac{\partial U(C_{t+2})}{\partial C_{t+2}}$$

To generate the desire to borrow in this model, it is assumed that $\beta < \frac{1}{1+r^w}$. This also implies, by conditions (3) and (4), that savings is optimally chosen to be zero (and remains zero for all equilibria studied). That savings only pays the world real interest rate reflects the assumption that international lenders are charging a risk premium to borrowers in this country but this is not translated into higher international returns to investment from this country. With zero savings we need only focus on conditions (4) and (5) to determine the steady state.

To maintain focus squarely on domestic borrowing behavior, international lenders are not explicitly modeled. To generate risk premia, an asymmetry of information is assumed to exist between domestic residents and international lenders over the exact per-date level of the endowment. Lenders are thought of as knowing a distribution, finitely bounded above and below, for domestic endowments, the mean of which is the actual, constant endowment. Accordingly, lenders charge risk premia based on the total amount of borrowing due on a given date.

A general form for interest rates that could reflect such lenders would then be[4]

$$(6) \qquad {}_t r_{t+z} = f(r^w, E_t\{{}_{t+z-1}B_{t+z|}\}, {}_t B_{t+z}) \qquad f_1, f_2, \text{ and } f_3 > 0$$

where , f_1,f_2, and f_3 are derivatives with respect to the first, second, and third arguments of the function, respectively. The expectations operator , E_t, has been included since the amount of the next date's new short term borrowing acquired (between dates t and $t+z$ when z = 2) is not known at time t. Lenders are assumed to form rational expectations over its level since the new short term debt due at date $t+z$ is relevant to the borrower's repayment ability at that date as well.

Leading (4) forward one date, substituting back into (4), and equating the result with condition (5), one obtains the requirement for all maturities to be held in equilibrium.

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[4] One can additionally add a time premium that makes longer term maturities more expensive than shorter term ones. While this would bias the results in favor of shorter maturities, it would also be intuitively appealing and have no substantial effect on the results as long as a reasonable premium was added (obviously one could include premia so high that long term maturities are never held which would affect the results without purpose). For a further exploration of various premia and information assumptions, see Ball (2003).

(7) $$(1+{}_t r_{t+1})(1+{}_{t+1} r_{t+2}) = (1+{}_t r_{t+2})^2$$

which is the condition that no arbitrage opportunities remain in equilibrium.

To solve the model analytically, a specific utility function must be specified. Letting utility be logarithmic suffices for our needs.

(8) $$u(C_t) \equiv \ln(C_t)$$

4. Model Dynamics

Using (8) with (4) and (5), the dynamics of the model can be written in the form of two difference equations resulting from the optimality conditions

(9) $$\frac{\Delta C_{t+1}}{C_t} = \beta(1+{}_t r_{t+1}) - 1$$

(10) $$\frac{\Delta C_{t+1}}{C_t} = \frac{\beta(1+{}_t r_{t+2})^2}{(1+{}_{t+1} r_{t+2})} - 1$$

Rewriting the flow budget constraint, (2), in difference equation form yields

(11) $$\Delta_t B_{t+1} + \Delta_t B_{t+2} = C_t - Y_t + {}_{t-1}r_t \bullet {}_{t-1}B_t + [(1+{}_{t-2}r_t)^2 - 1]_{t-2}B_t$$

where the following notational definitions have been used: $\Delta C_{t+1} \equiv C_{t+1} - C_t$ and $\Delta_t B_{t+1} \equiv {}_tB_{t+1} - {}_{t-1}B_t$ *and* $\Delta_t B_{t+2} \equiv {}_tB_{t+2} - {}_{t-2}B_t$. The dynamic system is completely defined by equations (9) through (11) and interest equation (6)[5].

These four equations, (9) through (11) and (6), allow one to draw two separate two dimensional phase diagrams (or one three dimensional diagram). One must then choose to take either short term or long term borrowing as exogenous. Since the intuition is more familiar in consumption-short-term-borrowing space, that is the one employed here.

Equation (9) defines the constant consumption locus and (11) the locus for constant short-term borrowing. Equation (6) is eliminated by substituting it into (9) and (11).

Since in steady state $\Delta C = \Delta_t B_{t+1} = \Delta_t B_{t+2} = 0$, time subscripts can be dropped although we must still distinguish between short and long term borrowing. Let an overscore denote steady state and a 1 or 2 denote short (one date) or long (two date) maturities, respectively. Equations (9) and (11) can be rewritten as

(9′) $$\beta(1+\overline{r}_1) = 1$$

and

(11′) $$\overline{C} = Y - \overline{r}_1\overline{B}_1 - [(1+\overline{r}_2)^2 - 1]\overline{B}_2$$

[5] Equation (9) comes from first differencing optimality condition (4) and imposing logarithmic utility. Equation (10) results from imposing logarithmic utility, combining optimality conditions (4) and (5) and then first differencing. Finally, equation (11) comes from rearranging terms in the flow budget constraint, (3), and setting savings to zero.

Equation (9′) is a standard Euler relation for steady state. It says that, in equilibrium, individuals cannot gain further by transferring consumption units from one date to another. This also means that individuals borrow until the interest rate equals the subjective rate of time preference. Graphically, equation (9′) indicates that the steady state level of borrowing is independent of the level of consumption and is thus a vertical line. The direction of the phase lines around the consumption locus are determined by the following derivative.

$$\frac{\partial \frac{\Delta C_{t+1}}{C_t}}{\partial_t B_{t+1}} = \beta \frac{\partial_t r_{t+1}}{\partial_t B_{t+1}} > 0 \tag{12}$$

which obtains because the interest rate is increasing in all its arguments and the subjective rate of time preference is strictly positive. Permanently increasing short term maturity holdings above the steady state level of borrowing leads to explosive, but unsustainable consumption through Ponzi financing. This is ruled out by the transversality condition (written in terms of the short-term asset)[6]:

$$\lim_{T\to\infty}\left(\frac{1}{\Pi_{v=t+1}^{s}(1+{}_{v-1}r_v)}\right)\left(B_{t+T+1}+B_{t+T+2}\right)=0 \tag{13}$$

[6] Imposing the transversality condition also allows one to write the lifetime budget constraint:

$$\sum_{s=t}^{\infty}\frac{Y_s - C_s}{\prod_{v=t+1}^{s}(1+{}_{v-1}r_v)} = (1+{}_{t-1}r_t)\,{}_{t-1}B_t + (1+{}_{t-2}r_t)^2\,{}_{t-2}B_t + \sum_{s=t}^{\infty}\frac{\prod_{v=t+2}^{s}(1+{}_{s-2}r_s)^2\,{}_{s-2}B_s}{\prod_{v=t+1}^{s}(1+{}_{v-1}r_v)}$$

Equation (11′) says that, in steady state, to maintain constant levels of short and long term borrowing, individual consumption will equal the constant endowment less per-date interest payments on both maturities. Individuals will constantly roll over their debt. It is important to note that this means steady state consumption is less than the endowment. Autarkic consumption is thus higher than the steady state level of consumption when borrowing occurs.

The incentive to default is strong here since doing so would allow individuals to permanently increase consumption to the level of their endowment. This paper does not explore the role of default and therefore rules it out. No-default equilibria can be supported by assuming large enough default penalties and a well functioning law enforcement structure.

Since long term borrowing is exogenous to the phase diagram, but an intricate part of this model, it is important to understand how changes in its level affect the $\Delta B_1 = 0$ locus, equation (11′).

(14)
$$\frac{d\overline{B}_1}{d\overline{B}_2} = -\frac{\dfrac{\partial \overline{r_1}}{\partial \overline{B}_2}}{\dfrac{\partial \overline{r_1}}{\partial \overline{B}_1}} < 0$$

The locus shifts left and associates a lower $\overline{C}$ with lower levels of $\overline{B}_1$. That this would be so comes from the no arbitrage condition in equilibrium and constant steady-state consumption. Individuals can only feasibly switch to longer term borrowing by decreasing the amount of short term assets they borrow. But given the present parameter values, an arbitrary reallocation of maturities is suboptimal, raises overall interest payments and thus lowers steady state consumption.

That (11′) is downward sloping can be seen from the following derivative.

$$(15) \qquad \frac{\partial \overline{C}}{\partial \overline{B}_1} = -\overline{r}_1 - \frac{\partial \overline{r}_1}{\partial \overline{B}_1}\overline{B}_1 - [(1+\overline{r}_2)^2 - 1] - 2(1+\overline{r}_2)\frac{\partial \overline{r}_2}{\partial \overline{B}_1}\overline{B}_2 < 0$$

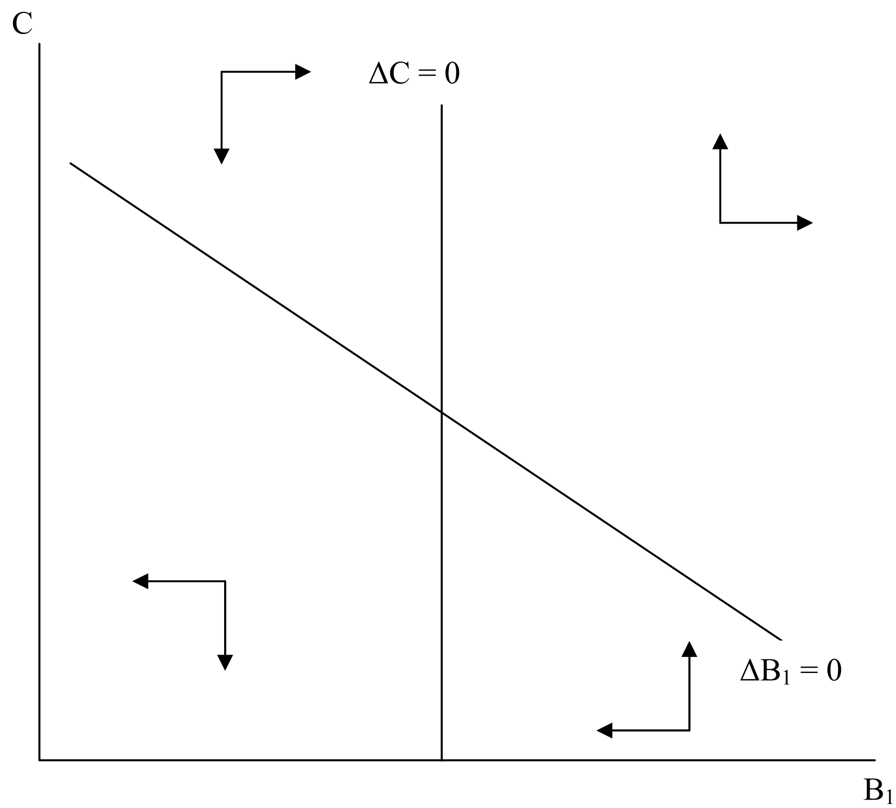
This obtains because interest rates and their derivatives are positive as are the levels of borrowing. In steady state, if you increase your level of borrowing, then your steady state consumption is lower because your per-date interest bill is permanently higher.

Again, because long term borrowing is exogenous to this diagram but relevant to the analysis, it is important to determine the direction (11′) moves when the level of long term borrowing changes. The steady state locus for short term borrowing intersects the C-axis where short term borrowing is zero. Imposing this condition on (12′) yields the intersection, which is below $C = Y$.

$$(16) \qquad \overline{C} = Y - [(1+\overline{r}_2)^2 - 1]\overline{B}_2$$

Clearly an increase in $\overline{B}_2$ lowers intercept consumption which means the locus has shifted leftward. The intuition is that, given a level of short term borrowing (here, zero), increases in long term steady state borrowing mean less residual endowment available for consumption.

Figure 1: Phase Diagram for The Dynamic System



The diagram reflects the loci and phase lines derived above. It is very tempting to include the saddle path here since one exists for this system holding $\overline{B}_2$ constant. The problem is that changes in B_1 do not leave B_2 constant. When B_2 is changes this will move the $\Delta B_1 = 0$ locus. Furthermore, one must interpret the directional phase lines as the initial direction of movement resulting from a shock, but before long-term borrowing has been affected. In this regard it is useful also to note that $\overline{B}_1$ is the amount of short term borrowing that exists at the beginning of a date. Nevertheless, the above diagram will be helpful in tracing out the effects of shocks in this model.

5. Experiments with the model – Sudden Stops

A sudden stop is simply the sudden cessation of new capital inflows into an economy. That is, a decrease in borrowing, resulting from an increase in risk premia charged to the economy in question. This could result from lender herding behavior leading flight from either a region of the world or a market type (like emerging market funds in general). Whatever the cause, it is important here that it is not driven by domestic fundamentals since in the present model the domestic fundamentals do not change. For simplicity it will be assumed that the increase in risk premia is large enough to force new borrowing to zero and that it is permanent. That is, the country is effectively cut off from world capital markets.

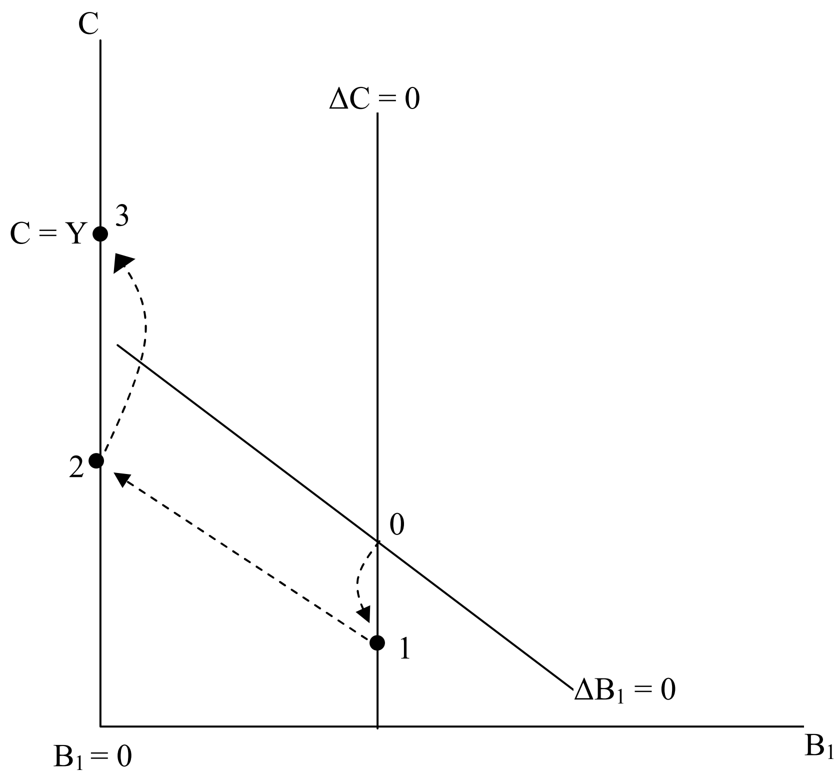
5.1 Unanticipated Complete Sudden Stops

The major difference between this and the previous case is that now the final resting point of the economy must display zero short and long term borrowing. When the sudden stop strikes there will be two dates during which the economy will adjust. The first is when the shock initially occurs. Call this date $t = 1$. At that date, individuals still have interest and principle repayments to finance.

$$C_1 = Y - (1+\overline{r}_1)\overline{B}_1 - (1+\overline{r}_2)^2 \overline{B}_2 \tag{17}$$

$\overline{B}_1$, $\overline{B}_2$, $\overline{r}_1$, and $\overline{r}_2$ are the predetermined steady state levels of borrowing and interest rates, respectively. Since no new borrowing can occur, this must be financed out of the current endowment, leaving less for consumption which consequently drops by the amount of repayment. Graphically this is a move from the old steady state at point 0 to point 1 in Figure 2.

Figure 2: Unanticipated Sudden Stop



At the next date, *t = 2*, consumption rises from its date 1 level because there is only predetermined long term borrowing plus interest to repay out of current endowment.

(18) $$C_1 < C_2 = Y - (1+\overline{r}_2)^2\,\overline{B}_2$$

Since the debt due at dates 1 and 2 are from the previous steady state, we know the amounts of short term and long term borrowing and that they are the same at both dates, by definition of the steady state. Since long term borrowing and its interest bill are the same at both dates, but there is no short term debt to repay, consumption at date 2 will be higher than at date 1. Graphically this occurs when there is no short term borrowing, but long term repayment remains. In Figure 2, this is point 2 on the C-axis. The transition from point 1 to 2 is *not* a saddle path. There is no saddle path for the sudden stop equilibrium since borrow remains at zero.

Finally, all predetermined borrowing plus interest has been repaid and the economy is at its new steady state, point 3, where per-date consumption equals the per-date constant endowment. While borrowing zero, technically individuals can save, allowing consumption to deviate from the endowment. This does not occur, however, since the world real interest rate and individual preferences have not changed.

5.2 Anticipated Sudden Stops

In this model a complete sudden stop means zero borrowing. Suppose that, at date $t = 1$, individuals learn a complete sudden stop will strike at date $t = 2$. They then face the following environment.

At $t = 1$, borrowing is still possible and the sudden stop is known to start at the next date. This is reflected in the budget constraint which becomes

$$Y + {}_1B_2 + {}_1B_3 = C_1 + (1+\bar{r}_1)\bar{B}_1 + (1+\bar{r}_2)^2\bar{B}_2 + {}_1S_2 \tag{19}$$

where $\bar{B}_1$, $\bar{B}_2$, $\bar{r}_1$, and $\bar{r}_2$ represent steady state levels of short term and long term borrowing and the associated short and long term interest rates, respectively. That consumption is subscripted and savings is included indicates that there may now be deviations from steady state levels due to the expected sudden stop.

At $t = 2$, the sudden stop is in effect and no new borrowing can occur.

(20) $$Y + (1 + r^w)\,{}_1S_2 = C_2 + (1 + {}_1r_2)\,{}_1B_2 + (1 + \overline{r}_2)^2\,\overline{B}_2 + {}_2S_3$$

where ${}_1B_2$ and ${}_1r_2$ reflect the level of short term borrowing chosen at the previous date and its cost, respectively. Savings is present because sudden stops don't affect the ability to save through international markets. Savings will be optimally set to zero. Finally, $\overline{B}_2$ and $\overline{r}_2$ are still the steady state levels of long term borrowing and interest.

At $t = 3$, no new borrowing can occur, but long term borrowing – chosen at date $t = 1$, must still be repaid. No steady state variables appear here.

(21) $$Y + (1 + r^w)\,{}_2S_3 = C_3 + (1 + {}_1r_3)^2\,{}_1B_3 + {}_3S_4$$

Finally, at $t = 4$, individuals begin their lives in the new sudden stop world where new borrowing is no longer feasible and there are no longer any remnants of borrowing from the previous steady state.

(22) $$Y + (1 + r^w)\,{}_3S_4 = C_4 + {}_4S_5$$

With savings set at zero, the new steady state is characterized by $C_t = Y_t$ for all t, which is the autarkic level of consumption.

For clarity's sake, it is also worth noting that the short term interest rate at date 2 reflects short term borrowing choices made at date 1 as well as the steady state long term borrowing chosen before the sudden stop was anticipated. The long term rate at date 3, on the other hand reflects only the long term borrowing chosen at date 1. That is, the interest rates are

$$(23) \qquad {}_1r_2 = f({}_1B_2, B_2) \qquad and \qquad {}_1r_3 = f({}_1B_3)$$

where B_L is the steady state level of long term borrowing. Also note that there is no longer an arbitrage condition similar to equation (8) because individuals no longer have the choice between borrowing long once or short twice.

Individuals can be thought of as solving a three period optimization problem, maximizing the following three date utility functional

$$(24) \qquad \sum_{t=1}^{3} \beta^{s-1} u(C_s)$$

subject to constraints (19), (20), and (21). Again, utility is time separable and logarithmic.

There are two relevant optimality conditions (savings is zero).

$$(25) \qquad C_2 = (1 + {}_1r_2)\beta C_1$$

and

(26) $$C_3 = (1 + {}_1r_3)^2 \beta^2 C_1$$

Together with the budget constraints, these constitute a system of five equations with five unknowns. Combining (25) with (20) and (26) with (22), and setting savings to zero yields

(27) $$C_1 = \frac{\tilde{Y}_2}{(1 + {}_1r_2)\beta} - \frac{1}{\beta}\,{}_1B_2 \qquad \textit{where } \tilde{Y}_2 \equiv Y - (1 + \bar{r}_2)^2 \bar{B}_2$$

which is date 1 consumption as a function of short term borrowing chosen in anticipation of the sudden stop. $\tilde{Y}_2$ has been defined as the endowment less predetermined payments (from the previous steady state) and is the individual's effective disposable endowment at date 2.

Similarly, combining (26) with (21) yields

(28) $$C_1 = \frac{Y}{(1 + {}_1r_3)^2 \beta^2} - \frac{1}{\beta^2}\,{}_1B_3$$

which is date 1 consumption as a function of long term borrowing chosen in anticipation of the sudden stop. That the endowment is not redefined here reflects that there is no longer any previous steady state borrowing due at date 3.

Using each of these in turn with the date 1 constraint, (19), yields the following two equations defining short and long term borrowing in terms of parameters and variables that are exogenous to this time frame.

(29) $$ {}_1B_3 = \frac{\tilde{Y}_2}{(1+{}_1r_2)\beta} - \tilde{Y}_1 - {}_1B_2\left(\frac{1+\beta}{\beta}\right) $$

(30) $$ {}_1B_2 = \frac{Y}{(1+{}_1r_3)^2\beta^2} - \tilde{Y}_1 - {}_1B_3\left(\frac{1+\beta^2}{\beta^2}\right) $$

where $\tilde{Y}_2$ is defined as before and $\tilde{Y}_1 \equiv Y - (1+\overline{r}_1)\overline{B}_1 - (1+\overline{r}_2)^2\overline{B}_2$, which is the effective disposable income at date 1. These are two equations in two unknowns and solvable, depending on the functional form of the interest rate.

Casual observation might suggest that, when such a shock is anticipated, individuals would not borrow money that will come due for repayment during the time of crisis. Without assuming a specific functional form for interest rates, it is still possible to show that individuals desire positive levels of both short and long term borrowing. This is also sufficient to show that savings will optimally be set to zero.

Suppose that there were no new borrowing so that ${}_1B_2 = {}_1B_3 = 0$. Then, ${}_1r_2 > {}_1r_3 = 0$ because ${}_1r_2$ depends on long term borrowing chosen in the previous steady state while ${}_1r_3$ depends only on new borrowing. By optimality condition (26), however, this would imply $C_3 = \beta^2 C_1$ which says $C_3 > C_1$ but this in turn implies the following, by combining (25) and (26), that

(31) $$C_3 = \frac{(1+{}_1r_3)^2\beta^2}{(1+{}_1r_2)\beta}C_2 = \frac{\beta C_2}{(1+{}_1r_2)} > C_1 \quad at\ {}_1B_2 = {}_1B_3 = 0$$

and, consequently,

(32) $$C_2 > \frac{(1+{}_1r_2)}{\beta}C_1$$

which violates optimality condition (25). For (25) and (26) to hold requires ${}_1B_2$ and ${}_1B_3$ to be positive[7].

6. Banning Short-Term Capital Flows

An often discussed proposal for mitigating the damage from sudden stops is to impose a restriction on short term capital flows[8]. In particular there seems to be a belief that encouraging a maturity shift toward longer term maturities would either reduce the likelihood of a sudden stop or improve welfare during one. Since the sudden stop here is exogenous, we can only evaluate the welfare claim in this model.

To check the strongest case this section briefly considers banning short term capital altogether, leaving individuals with long term maturities only. That is, ${}_tB_{t+1}$ is set to zero in this model.

The budget constraint, (2), becomes

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[7] That they can't be negative reflects that interest rates increase with borrowing and have a lower bound at the riskless world interest rate. This is also the reason savings must be zero.

[8] See Feldstein (1999), Rodrik and Velasco (1999), or Mussa (2000) for well cited arguments along this line.

$$Y_t + (1+r^w)\, {}_{t-1}S_t + {}_tB_{t+2} = C_t + (1+ {}_{t-2}r_t)^2\, {}_{t-2}B_t + {}_tS_{t+1} \tag{33}$$

and the only two relevant optimality conditions are (3) and (5) referring to savings and long term borrowing respectively. Since preferences relative to world interest rates are unchanged, savings is optimally kept at zero. Accordingly, the steady state is defined by first-differencing (5) and setting it equal to zero as well as writing (33) in terms of differences and setting to zero. The following two equations emerge.

$$1 = (1+\bar{r}_2)^2 \beta^2 \qquad \textit{with } \bar{r}_2 = f(r^w, \bar{B}_2) \tag{34}$$

and

$$0 = Y - C - (1+\bar{r}_2)^2 \bar{B}_2 \tag{35}$$

A phase diagram in terms of consumption and long-term borrowing can be drawn in a fashion similar to Figure 2. The locus of steady state consumption, from (34), is vertical and (35) results in a downward sloping steady state long term borrowing locus as well. The phase lines and saddle path are diagrammatically the same as before.

The biggest change is that, without short term borrowing, interest rates depend only on the new borrowing contracted since this is the only amount that will be due on the repayment date. This is the only substantive change. All the lessons from the analysis of the shocks can be applied here as well. There is a transition during complete sudden stops that takes two dates to complete since predetermined long term borrowing shows up in the current and subsequent date. After that, individuals live in an autarkic environment.

It is clear that individuals are worse off here than if long-term borrowing were banned instead (i.e., ${}_tB_{t+2} = 0$ for all t). There are two main reasons for this. First, the length of the transition to the autarkic steady state would be shorter. With long-term borrowing present, when a sudden stop strikes, consumption falls radically, lowering utility, and then rises to the sudden stop steady state ($C = Y$) over two dates. That is, for two dates it is below the new steady state and for one of those dates, much lower. The pain comes from the transition from the borrowing steady state to the sudden stop one[9]. The presence of long term borrowing extends the length of this painful process.

Second, the multiple maturity steady state is preferred to the single maturity one. If this were not the case, the basic model where both are present would result in a corner solution where only one maturity would be chosen. That this is not the case and that both are held implies individuals value both. The reason is that they are able to spread the cost of borrowing over more dates than with a single maturity type. This lowers their overall interest bill and better allows them to smooth consumption.

7. Conclusion

Much of the sudden stop literature focuses on their causes, monetary effects, financial crises and relative price shocks. Most of the models developed contain short term borrowing only or include different maturities in a Diamond-Dybvig setup where

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[9] Similarly, all the initial (non-shock) gains come in the transition to steady state when opening the economy from autarky. Individual preferences are such that they prefer to borrow and consider the early days of consuming future endowment early worth the cost of spending the rest of their lives at a lower consumption steady state.

individuals aren't really smoothing per-date lifetime consumption. Nevertheless, many researchers and policy makers conclude – sometimes parenthetically, sometimes not – that short-term borrowing is partially to blame for the damage from sudden stops and that there is room to discuss capital controls to encourage longer term borrowing.

This paper presents a basic representative agent model that includes both short and long term borrowing in an infinite horizon framework. Doing so allows one to ask steady state questions not possible in three date Diamond-Dybvig based models when multiple maturities are present.

This is intended as a first step forward in addressing multiple maturity related issues in a richer framework that is consistent with modern international macroeconomics. Accordingly, the notation and conceptual framework of this model is intentionally consistent with most of the models presented in Obstfeld and Rogoff (1996). The hope is that future research will better be able to handle multiple maturities in a way that is consistent with much of the current thinking in this field.

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